Metric Temporal Answer Set Programming over Timed Traces*

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Abstract. In temporal extensions of Answer Set Programming (ASP) based on linear-time, the behavior of dynamic systems is captured by sequences of states. While this representation reflects their relative order, it abstracts away the specific times associated with each state. In many applications, however, timing constraints are important like, for instance, when planning and scheduling go hand in hand. We address this by developing a metric extension of linear-time temporal equilibrium logic, in which temporal operators are constrained by intervals over natural numbers. The resulting Metric Equilibrium Logic provides the foundation of an ASP-based approach for specifying qualitative and quantitative dynamic constraints. To this end, we define a translation of metric formulas into monadic first-order formulas and give a correspondence between their models in Metric Equilibrium Logic and Monadic Quantified Equilibrium Logic, respectively. Interestingly, our translation provides a blue print for implementation in terms of ASP modulo difference constraints.

1 Introduction

Reasoning about action and change, or more generally about dynamic systems, is not only central to knowledge representation and reasoning but at the heart of computer science [14]. In practice, this often requires both qualitative as well as quantitative dynamic constraints. For instance, when planning and scheduling at once, actions may have durations and their effects may need to meet deadlines.

Over the last years, we addressed qualitative dynamic constraints by combining traditional approaches, like Dynamic and Linear Temporal Logic (DL [16] and LTL [26]), with the base logic of Answer Set Programming (ASP [21]), namely, the logic of Here-and-There (HT [17]) and its non-monotonic extension, called Equilibrium Logic [24]. This resulted in non-monotonic linear dynamic and temporal equilibrium logics (DEL [5, 8] and TEL [1, 11]) that gave rise to the temporal ASP system *telingo* [10, 7] extending the ASP system *clingo* [15].

Another commonality of dynamic and temporal logics is that they abstract from specific time points when capturing temporal relationships. For instance, in temporal logic, we can use the formula $\square(use \rightarrow \Diamond clean)$ to express that a

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machine has to be eventually cleaned after being used. Nothing can be said about the delay between using and cleaning the machine.

A key design decision was to base both logics, TEL and DEL, on the same linear-time semantics. We continued to maintain the same linear-time semantics, embodied by sequences of states, when elaborating upon a first "light-weight" metric temporal extension of HT [9]. The "light-weightiness" is due to treating time as a state counter by identifying the next time with the next state. For instance, this allows us to refine our example by stating that, if the machine is used, it has to be cleaned within the next 3 states, viz. $\Box(use \rightarrow \Diamond_{[1..3]} clean)$. Although this permits the restriction of temporal operators to subsequences of states, no fine-grained timing constraints are expressible.

In this paper, we address this by associating each state with its *time*, as done in Metric Temporal Logic (MTL [20]). This allows us to measure time differences between events. For instance, in our example, we may thus express that whenever the machine is used, it has to be cleaned within 60 to 120 time units, by writing:

$$\Box(use \rightarrow \Diamond_{[60..120]} clean) \ .$$

Unlike the non-metric version, this stipulates that once *use* is true in a state, *clean* must be true in some future state whose associated time is at least 60 and at most 120 time units after the time of *use*. The choice of time domain is crucial, and might even lead to undecidability in the continuous case. We rather adapt a discrete approach that offers a sequence of snapshots of a dynamic system.

2 Metric temporal logic

Given $m \in \mathbb{N}$ and $n \in \mathbb{N} \cup \{\omega\}$, we let $[m..n]$ stand for the set $\{i \in \mathbb{N} \mid m \leq i \leq n\}$, $[m..n)$ for $\{i \in \mathbb{N} \mid m \leq i < n\}$, and $(m..n]$ stand for $\{i \in \mathbb{N} \mid m < i \leq n\}$.

Given a set $\mathcal{A}$ of propositional variables (called *alphabet*), a *metric formula* φ is defined by the grammar:

$$\varphi ::= p \mid \bot \mid \varphi_1 \otimes \varphi_2 \mid \bullet_{\mathcal{I}} \varphi \mid \varphi_1 \, \mathbf{S}_{\mathcal{I}} \, \varphi_2 \mid \varphi_1 \, \mathbf{T}_{\mathcal{I}} \, \varphi_2 \mid \circ_{\mathcal{I}} \varphi \mid \varphi_1 \, \mathbf{U}_{\mathcal{I}} \, \varphi_2 \mid \varphi_1 \, \mathbf{R}_{\mathcal{I}} \, \varphi_2$$

where $p \in \mathcal{A}$ is an atom and $\otimes$ is any binary Boolean connective $\otimes \in \{\rightarrow, \wedge, \vee\}$. The last six cases above correspond to temporal operators, each of them indexed by some interval $\mathcal{I}$ of the form $[m..n)$ with $m \in \mathbb{N}$ and $n \in \mathbb{N} \cup \{\omega\}$. In words, $\bullet_{\mathcal{I}}$, $\mathbf{S}_{\mathcal{I}}$, and $\mathbf{T}_{\mathcal{I}}$ are past operators called *previous*, *since*, and *trigger*, respectively; their future counterparts $\circ_{\mathcal{I}}$, $\mathbf{U}_{\mathcal{I}}$, and $\mathbf{R}_{\mathcal{I}}$ are called *next*, *until*, and *release*. We let subindex $[m..n]$ stand for $[m..n{+}1)$, provided $n \neq \omega$. Also, we sometimes use the subindices '$\leq n$', '$\geq m$' and 'm' as abbreviations of intervals $[0..n]$, $[m..\omega)$ and $[m..m]$, respectively. Also, whenever $\mathcal{I} = [0..\omega)$, we simply omit subindex $\mathcal{I}$.

A *metric theory* is a (possibly infinite) set of metric formulas.

We also define several common derived operators like the Boolean connectives $\top \stackrel{def}{=} \neg\bot$, $\neg\varphi \stackrel{def}{=} \varphi \rightarrow \bot$, $\varphi \leftrightarrow \psi \stackrel{def}{=} (\varphi \rightarrow \psi) \wedge (\psi \rightarrow \varphi)$, and the following temporal operators:

$\blacksquare_{\mathcal{I}}\varphi \stackrel{def}{=} \bot\, \mathbf{T}_{\mathcal{I}}\, \varphi$	*always before*	$\square_{\mathcal{I}}\varphi \stackrel{def}{=} \bot\, \mathbf{R}_{\mathcal{I}}\, \varphi$	*always afterward*
$\blacklozenge_{\mathcal{I}}\varphi \stackrel{def}{=} \top\, \mathbf{S}_{\mathcal{I}}\, \varphi$	*eventually before*	$\lozenge_{\mathcal{I}}\varphi \stackrel{def}{=} \top\, \mathbf{U}_{\mathcal{I}}\, \varphi$	*eventually afterward*
$\mathbf{I} \stackrel{def}{=} \neg\bullet\top$	*initial*	$\mathbf{F} \stackrel{def}{=} \neg\circ\top$	*final*
$\widehat{\bullet}_{\mathcal{I}}\varphi \stackrel{def}{=} \bullet_{\mathcal{I}}\varphi \vee \neg\bullet_{\mathcal{I}}\top$	*weak previous*	$\widehat{\circ}_{\mathcal{I}}\varphi \stackrel{def}{=} \circ_{\mathcal{I}}\varphi \vee \neg\circ_{\mathcal{I}}\top$	*weak next*

Note that *initial* and *final* are not indexed by any interval; they only depend on the state of the trace, not on the actual time that this state is mapped to. On the other hand, the weak version of *next* can no longer be defined in terms of *final*, as done in [11] with non-metric $\widehat{\circ}\varphi \equiv \circ\varphi \vee \mathbf{F}$. For the metric case $\widehat{\circ}_{\mathcal{I}}\varphi$, the disjunction $\circ_{\mathcal{I}}\varphi \vee \neg\circ_{\mathcal{I}}\top$ must be used instead, in order to keep the usual dualities among operators (the same applies to weak *previous*).

The definition of *Metric Equilibrium Logic* (MEL for short) is done in two steps. We start with the definition of a monotonic logic called *Metric logic of Here-and-There* (MHT), a temporal extension of the intermediate logic of Here-and-There [17]. We then select some models from MHT that are said to be in equilibrium, obtaining in this way a non-monotonic entailment relation.

An example of metric formulas is the modeling of traffic lights. While the light is red by default, it changes to green within less than 15 time units (say, seconds) whenever the button is pushed; and it stays green for another 30 seconds at most. This can be represented as follows.

$$\square(\mathit{red} \wedge \mathit{green} \rightarrow \bot) \tag{1}$$

$$\square(\neg \mathit{green} \rightarrow \mathit{red}) \tag{2}$$

$$\square\big(\mathit{push} \rightarrow \lozenge_{[1..15)}(\square_{\leq 30}\ \mathit{green})\big) \tag{3}$$

Note that this example combines a default rule (2) with a metric rule (3), describing the initiation and duration period of events. This nicely illustrates the interest in non-monotonic metric representation and reasoning methods.

A *Here-and-There trace* (for short HT-*trace*) of length $\lambda \in \mathbb{N} \cup \{\omega\}$ over alphabet $\mathcal{A}$ is a sequence of pairs $(\langle H_i, T_i\rangle)_{i\in[0..\lambda)}$ with $H_i \subseteq T_i \subseteq \mathcal{A}$ for any $i \in [0..\lambda)$. For convenience, we usually represent an HT-trace as the pair $\langle \mathbf{H}, \mathbf{T}\rangle$ of traces $\mathbf{H} = (H_i)_{i\in[0..\lambda)}$ and $\mathbf{T} = (T_i)_{i\in[0..\lambda)}$. Notice that, when $\lambda = \omega$, this covers traces of infinite length. We say that $\langle \mathbf{H}, \mathbf{T}\rangle$ is *total* when $\mathbf{H} = \mathbf{T}$, that is, $H_i = T_i$ for all $i \in [0..\lambda)$.

Definition 1. *A* timed trace $(\langle \mathbf{H}, \mathbf{T}\rangle, \tau)$ *over* $(\mathbb{N}, <)$ *is a pair consisting of*

- *an* HT-*trace* $\langle \mathbf{H}, \mathbf{T}\rangle = (\langle H_i, T_i\rangle)_{i\in[0..\lambda)}$ *and*
- *a function* $\tau : [0..\lambda) \rightarrow \mathbb{N}$ *such that* $\tau(i) \leq \tau(i{+}1)$.

A timed trace of length $\lambda > 1$ *is called* strict *if* $\tau(i) < \tau(i{+}1)$ *for all* $i \in [0..\lambda)$ *such that* $i + 1 < \lambda$ *and* non-strict *otherwise. We assume w.l.o.g. that* $\tau(0) = 0$. □

Function τ assigns, to each state index $i \in [0..\lambda)$, a time point $\tau(i) \in \mathbb{N}$ representing the number of time units (seconds, miliseconds, etc, depending on the chosen granularity) elapsed since time point $\tau(0) = 0$ chosen as the beginning of the trace. The difference to the variant of MHT presented in [9] boils down to the choice of function τ. In [9], this was the identity function on the interval $[0..\lambda)$.

Given any timed HT-trace, satisfaction of formulas is defined as follows.

Definition 2 (MHT-**satisfaction).** *A timed* HT-*trace* $\mathbf{M} = (\langle \mathbf{H}, \mathbf{T} \rangle, \tau)$ *of length* λ *over alphabet* $\mathcal{A}$ satisfies *a metric formula* φ *at step* $k \in [0..\lambda)$*, written* $\mathbf{M}, k \models \varphi$*, if the following conditions hold:*

1. $\mathbf{M}, k \not\models \bot$
2. $\mathbf{M}, k \models p$ *if* $p \in H_k$ *for any atom* $p \in \mathcal{A}$
3. $\mathbf{M}, k \models \varphi \wedge \psi$ *iff* $\mathbf{M}, k \models \varphi$ *and* $\mathbf{M}, k \models \psi$
4. $\mathbf{M}, k \models \varphi \vee \psi$ *iff* $\mathbf{M}, k \models \varphi$ *or* $\mathbf{M}, k \models \psi$
5. $\mathbf{M}, k \models \varphi \rightarrow \psi$ *iff* $\mathbf{M}', k \not\models \varphi$ *or* $\mathbf{M}', k \models \psi$*, for both* $\mathbf{M}' = \mathbf{M}$ *and* $\mathbf{M}' = (\langle \mathbf{T}, \mathbf{T} \rangle, \tau)$
6. $\mathbf{M}, k \models \bullet_{\mathcal{I}} \varphi$ *iff* $k > 0$ *and* $\mathbf{M}, k{-}1 \models \varphi$ *and* $\tau(k) - \tau(k{-}1) \in \mathcal{I}$
7. $\mathbf{M}, k \models \varphi \, \mathbf{S}_{\mathcal{I}} \, \psi$ *iff for some* $j \in [0..k]$ *with* $\tau(k) - \tau(j) \in \mathcal{I}$*, we have* $\mathbf{M}, j \models \psi$ *and* $\mathbf{M}, i \models \varphi$ *for all* $i \in (j..k]$
8. $\mathbf{M}, k \models \varphi \, \mathbf{T}_{\mathcal{I}} \, \psi$ *iff for all* $j \in [0..k]$ *with* $\tau(k) - \tau(j) \in \mathcal{I}$*, we have* $\mathbf{M}, j \models \psi$ *or* $\mathbf{M}, i \models \varphi$ *for some* $i \in (j..k]$
9. $\mathbf{M}, k \models \circ_{\mathcal{I}} \varphi$ *iff* $k + 1 < \lambda$ *and* $\mathbf{M}, k{+}1 \models \varphi$ *and* $\tau(k{+}1) - \tau(k) \in \mathcal{I}$
10. $\mathbf{M}, k \models \varphi \, \mathbf{U}_{\mathcal{I}} \, \psi$ *iff for some* $j \in [k..\lambda)$ *with* $\tau(j) - \tau(k) \in \mathcal{I}$*, we have* $\mathbf{M}, j \models \psi$ *and* $\mathbf{M}, i \models \varphi$ *for all* $i \in [k..j)$
11. $\mathbf{M}, k \models \varphi \, \mathbf{R}_{\mathcal{I}} \, \psi$ *iff for all* $j \in [k..\lambda)$ *with* $\tau(j) - \tau(k) \in \mathcal{I}$*, we have* $\mathbf{M}, j \models \psi$ *or* $\mathbf{M}, i \models \varphi$ *for some* $i \in [k..j)$ □

Satisfaction of derived operators can be easily deduced:

Proposition 1. *Let* $\mathbf{M} = (\langle \mathbf{H}, \mathbf{T} \rangle, \tau)$ *be a timed* HT-*trace of length* λ *over* $\mathcal{A}$*. Given the respective definitions of derived operators, we get the following satisfaction conditions:*

13. $\mathbf{M}, k \models \mathbf{I}$ *iff* $k = 0$
14. $\mathbf{M}, k \models \widehat{\bullet}_{\mathcal{I}} \varphi$ *iff* $k = 0$ *or* $\mathbf{M}, k{-}1 \models \varphi$ *or* $\tau(k) - \tau(k{-}1) \notin \mathcal{I}$
15. $\mathbf{M}, k \models \blacklozenge_{\mathcal{I}} \varphi$ *iff* $\mathbf{M}, i \models \varphi$ *for some* $i \in [0..k]$ *with* $\tau(k) - \tau(i) \in \mathcal{I}$
16. $\mathbf{M}, k \models \blacksquare_{\mathcal{I}} \varphi$ *iff* $\mathbf{M}, i \models \varphi$ *for all* $i \in [0..k]$ *with* $\tau(k) - \tau(i) \in \mathcal{I}$
17. $\mathbf{M}, k \models \mathbf{F}$ *iff* $k + 1 = \lambda$
18. $\mathbf{M}, k \models \widehat{\circ}_{\mathcal{I}} \varphi$ *iff* $k + 1 < \lambda$ *or* $\mathbf{M}, k{+}1 \models \varphi$ *or* $\tau(k{+}1) - \tau(k) \notin \mathcal{I}$
19. $\mathbf{M}, k \models \Diamond_{\mathcal{I}} \varphi$ *iff* $\mathbf{M}, i \models \varphi$ *for some* $i \in [k..\lambda)$ *with* $\tau(i) - \tau(k) \in \mathcal{I}$
20. $\mathbf{M}, k \models \Box_{\mathcal{I}} \varphi$ *iff* $\mathbf{M}, i \models \varphi$ *for all* $i \in [k..\lambda)$ *with* $\tau(i) - \tau(k) \in \mathcal{I}$ □

A formula φ is a *tautology* (or is valid), written $\models \varphi$, iff $\mathbf{M}, k \models \varphi$ for any timed HT-trace $\mathbf{M}$ and any $k \in [0..\lambda)$. MHT is the logic induced by the set of all such tautologies. For two formulas φ, ψ we write $\varphi \equiv \psi$, iff $\models \varphi \leftrightarrow \psi$, that is, $\mathbf{M}, k \models \varphi \leftrightarrow \psi$ for any timed HT-trace $\mathbf{M}$ of length λ and any $k \in [0..\lambda)$. A timed HT-trace $\mathbf{M}$ is an MHT *model* of a metric theory Γ if $\mathbf{M}, 0 \models \varphi$ for all $\varphi \in \Gamma$. The set of MHT models of Γ having length λ is denoted as $\mathrm{MHT}(\Gamma, \lambda)$, whereas $\mathrm{MHT}(\Gamma) \stackrel{def}{=} \bigcup_{\lambda=0}^{\omega} \mathrm{MHT}(\Gamma, \lambda)$ is the set of all MHT models of Γ of any length. We may obtain fragments of any metric logic by imposing restrictions on the timed traces used for defining tautologies and models. That is, MHT_f stands for the restriction of MHT to traces of any finite length $\lambda \in \mathbb{N}$ and MHT_ω corresponds to the restriction to traces of infinite length $\lambda = \omega$.

An interesting subset of MHT is the one formed by total timed traces $(\langle \mathbf{T}, \mathbf{T} \rangle, \tau)$. In the non-metric version of temporal HT, the restriction to total models corresponds to Linear Temporal Logic (LTL [26]). In our case, the restriction to total traces defines a metric version of LTL, that we call *Metric Temporal Logic* (MTL for short). It can be proved that MTL are those models of MHT satisfying the excluded middle axiom schema: $\square(p \vee \neg p)$ for any atom $p \in \mathcal{A}$. We present next several properties about total traces and the relation between MHT and MTL.

Proposition 2 (Persistence). *Let $(\langle \mathbf{H}, \mathbf{T} \rangle, \tau)$ be a timed* HT*-trace of length λ over $\mathcal{A}$ and let φ be a metric formula over $\mathcal{A}$. Then, for any $k \in [0..\lambda)$, if $(\langle \mathbf{H}, \mathbf{T} \rangle, \tau), k \models \varphi$ then $(\langle \mathbf{T}, \mathbf{T} \rangle, \tau), k \models \varphi$.* □

Thanks to Proposition 2 and a decidability result in [23], we get:

Corollary 1 (Decidability of MHT_f). *The logic of* MHT_f *is decidable.* □

Proposition 3. *Let $(\langle \mathbf{H}, \mathbf{T} \rangle, \tau)$ be a timed* HT*-trace of length λ over $\mathcal{A}$ and let φ be a metric formula over $\mathcal{A}$. Then, $(\langle \mathbf{H}, \mathbf{T} \rangle, \tau), k \models \neg\varphi$ iff $(\langle \mathbf{T}, \mathbf{T} \rangle, \tau), k \not\models \varphi$.* □

Proposition 4. *Let φ and ψ be metric formulas without implication (and so, without negation either). Then, $\varphi \equiv \psi$ in* MTL *iff $\varphi \equiv \psi$ in* MHT*.* □

Many tautologies in MHT or its fragments have a dual version depending on the nature of the operators involved. The following pair of duality properties allows us to save space and proof effort when listing interesting valid equivalences. We define all pairs of dual connectives as follows: $\wedge_\mathcal{I}/\vee_\mathcal{I}$, $\top_\mathcal{I}/\bot_\mathcal{I}$, $\mathbf{U}_\mathcal{I}/\mathbf{R}_\mathcal{I}$, $\circ_\mathcal{I}/\widehat{\circ}_\mathcal{I}$, $\square_\mathcal{I}/\Diamond_\mathcal{I}$, $\mathbf{S}_\mathcal{I}/\mathbf{T}_\mathcal{I}$, $\bullet_\mathcal{I}/\widehat{\bullet}_\mathcal{I}$, $\blacksquare_\mathcal{I}/\blacklozenge_\mathcal{I}$. For any formula φ without implications, we define $\delta(\varphi)$ as the result of replacing each connective by its dual operator.

Then, we get the following corollary of Proposition 4.

Corollary 2 (Boolean Duality). *Let φ and ψ be formulas without implication. Then,* MHT *satisfies: $\varphi \equiv \psi$ iff $\delta(\varphi) \equiv \delta(\psi)$.* □

Let $\mathbf{U}_\mathcal{I}/\mathbf{S}_\mathcal{I}$, $\mathbf{R}_\mathcal{I}/\mathbf{T}_\mathcal{I}$, $\circ_\mathcal{I}/\bullet_\mathcal{I}$, $\widehat{\circ}_\mathcal{I}/\widehat{\bullet}_\mathcal{I}$, $\square_\mathcal{I}/\blacksquare_\mathcal{I}$, and $\Diamond_\mathcal{I}/\blacklozenge_\mathcal{I}$ be all pairs of swapped-time connectives and $\sigma(\varphi)$ be the replacement in φ of each connective by its swapped-time version. Then, we have the following result for finite traces.

Lemma 1. *There exists a mapping ϱ on finite timed* HT*-traces $\mathbf{M}$ of the same length $\lambda \geq 0$ such that for any $k \in [0..\lambda)$, $\mathbf{M}, k \models \varphi$ iff $\varrho(\mathbf{M}), \lambda - 1 - k \models \sigma(\varphi)$.*

Theorem 1 (Temporal Duality Theorem). *A metric formula φ is a MHT_f-tautology iff $\sigma(\varphi)$ is a MHT_f-tautology.* □

As in traditional Equilibrium Logic [24], non-monotonicity is achieved by a selection among the MHT models of a theory.

Definition 3 (Metric Equilibrium/Stable Model). *Let $\mathfrak{S}$ be some set of timed* HT-*traces. A total timed* HT-*trace* $(\langle \mathbf{T}, \mathbf{T} \rangle, \tau) \in \mathfrak{S}$ *is a metric equilibrium model of* $\mathfrak{S}$ *iff there is no other* $\mathbf{H} < \mathbf{T}$ *such that* $(\langle \mathbf{H}, \mathbf{T} \rangle, \tau) \in \mathfrak{S}$*. The timed trace* $(\mathbf{T}, \tau)$ *is called a metric stable model of* $\mathfrak{S}$. □

We talk about metric equilibrium (or metric stable) models of a theory Γ when $\mathfrak{S} = \mathrm{MHT}(\Gamma)$, and we write $\mathrm{MEL}(\Gamma, \lambda)$ and $\mathrm{MEL}(\Gamma)$ to stand for the metric equilibrium models of $\mathrm{MHT}(\Gamma, \lambda)$ and $\mathrm{MHT}(\Gamma)$, respectively. *Metric Equilibrium Logic* (MEL) is the non-monotonic logic induced by the metric equilibrium models of metric theories. As before, variants MEL_f and MEL_ω refer to MEL when restricted to traces of finite and infinite length, respectively.

Proposition 5. *The set of metric equilibrium models of Γ can be partitioned on the trace lengths, namely,* $\bigcup_{\lambda=0}^{\omega} \mathrm{MEL}(\Gamma, \lambda) = \mathrm{MEL}(\Gamma)$. □

We can enforce metric models to be traces with a strict timing function τ, that is, $\tau(i) < \tau(i+1)$ for any i such that $i+1 \in [1..\lambda)$. This can be achieved with the simple addition of the axiom $\Box \neg \circ_0 \top$. In the following, we assume that this axiom is included and consider, in this way, strict timing. For instance, a consequence of strict timing is that one-step operators become definable in terms of other connectives. For non-empty intervals $[m..n)$ with $m < n$, we get:

$$\bullet_{[m..n)} \varphi \equiv \blacksquare_{[1..m)} \bot \wedge \blacklozenge_{[h..n)} \varphi$$
$$\circ_{[m..n)} \varphi \equiv \Box_{[1..m)} \bot \wedge \Diamond_{[h..n)} \varphi \qquad \text{where } h = \max(1, m);$$

whereas for empty intervals with $m \geq n$, we obtain $\bullet_{[m..n)} \varphi \equiv \circ_{[m..n)} \varphi \equiv \bot$.

Back to our example, suppose we have the theory Γ consisting of the formulas (1)-(3). In the example, we abbreviate subsets of the set of atoms $\{green, push, red\}$ as strings formed by their initials: For instance, *pr* stands for $\{push, red\}$. For readability sake, we represent traces (T_0, T_1, T_2) as $T_0 \cdot T_1 \cdot T_2$. Consider first the total models of Γ: the first two rules force one of the two atoms *green* or *red* to hold at every state. Besides, we can choose adding *push* or not, but if we do so, *green* should hold later on according to (3). Now, for any total model $(\langle \mathbf{T}, \mathbf{T} \rangle, \tau), 0 \models \Gamma$ where *green* or *push* hold at some states, we can always form $\mathbf{H}$ where we remove those atoms from all the states and it is not difficult to see that $(\langle \mathbf{H}, \mathbf{T} \rangle, \tau), 0 \models \Gamma$, so $(\langle \mathbf{T}, \mathbf{T} \rangle, \tau)$ is not in equilibrium. As a consequence, metric equilibrium models of Γ have the form $(\langle \mathbf{T}, \mathbf{T} \rangle, \tau)$ being $\mathbf{T} = \langle T_i \rangle_{i \in [0..\lambda)}$ with $T_i = \{red\}$ for all $i \in [0..\lambda)$ and any arbitrary strict timing function τ. To illustrate non-monotonicity, suppose now that we have $\Gamma' = \Gamma \cup \{\circ_5\, push\}$ and, for simplicity, consider length $\lambda = 3$ and traces of the form $T_0 \cdot T_1 \cdot T_2$. Again, it is not hard to see that total models with *green* or *push* in state T_0 are not in equilibrium, being the only option $T_0 = \{red\}$. The same happens for *green* at T_1, so we get $T_1 = \{push, red\}$ as only candidate for equilibrium model. However, since $push \in T_1$, the only possibility to satisfy the consequent of (3) is having $green \in T_2$. Again, we can also see that adding *push* at that state would not be in equilibrium so that the only trace in equilibrium is $T_0 = \{red\}$, $T_1 = \{push, red\}$ and $T_2 = \{green\}$. As for the timing, $\tau(0) = 0$ is fixed, and satisfaction of formula

($\circ_5$ *push*) fixes $\tau(1) = 5$. Then, from (3) we conclude that *green* must hold at any moment starting at t between $5+1$ and $5+14$ and is kept true in all states between t and $t+30$ time units, but as $\lambda = 2$, this means just t. To sum up, we get 14 metric equilibrium models with $\tau(0) = 0$ and $\tau(1) = 5$ fixed, but varying $\tau(2)$ between 6 and 19.

We observe next the effect of the semantics of *always* and *eventually* on truth constants. Let φ be an arbitrary metric formula and $m, n \in \mathbb{N}$. Then, $\Box_{[m..n)}\bot$ means that there is no state in interval $[m..n)$ and $\Diamond_{[m..n)}\top$ means that there is at least one state in this interval. The formula $\Box_{[m..n)}\top$ is a tautology, whereas $\Diamond_{[m..n)}\bot$ is unsatisfiable. The same applies to past operators $\blacklozenge_{[m..n)}$ and $\blacksquare_{[m..n)}$.

The following equivalences state that interval $\mathcal{I} = [0..0]$ makes all binary metric operators collapse into their right hand argument formula, whereas unary operators collapse to a truth constant. For metric formulas ψ and φ, we have:

$$\psi \, \mathbf{U}_0 \, \varphi \equiv \psi \, \mathbf{R}_0 \, \varphi \equiv \varphi \tag{4}$$

$$\circ_0 \, \varphi \equiv \bullet_0 \, \varphi \equiv \bot \tag{5}$$

$$\widehat{\circ}_0 \, \varphi \equiv \widehat{\bullet}_0 \, \varphi \equiv \top \tag{6}$$

The last two lines are precisely an effect of dealing with strict traces: For instance, $\circ_0 \, \varphi \equiv \bot$ tells us that it is always impossible to have a successor state with the same time (the time difference is 0) as the current one, regardless of the formula φ we want to check. The next lemma allows us to unfold metric operators for single-point time intervals $[n..n]$ with $n > 0$.

Lemma 2. *For metric formulas ψ and φ and for $n > 0$, we have:*

$$\psi \, \mathbf{U}_n \, \varphi \equiv \bigvee_{i=1}^{n} \circ_i(\psi \, \mathbf{U}_{n-i} \, \varphi) \tag{7}$$

$$\psi \, \mathbf{R}_n \, \varphi \equiv \bigwedge_{i=1}^{n} \widehat{\circ}_i(\psi \, \mathbf{R}_{n-i} \, \varphi) \tag{8}$$

$$\Diamond_n \varphi \equiv \bigvee_{i=1}^{n} \circ_i \Diamond_{n-i} \varphi \tag{9}$$

$$\Box_n \varphi \equiv \bigwedge_{i=1}^{n} \widehat{\circ}_i \Box_{n-i} \varphi \tag{10}$$

The same applies for the dual past operators. □

Going one step further, we can also unfold *until* and *release* for intervals of the form $[0..n]$ with the application of the following result.

Lemma 3. *For metric formulas ψ and φ and for $n > 0$, we have:*

$$\psi \, \mathbf{U}_{\leq n} \, \varphi \equiv \varphi \vee (\psi \wedge \bigvee_{i=1}^{n} \circ_i(\psi \, \mathbf{U}_{\leq (n-i)} \, \varphi)) \tag{11}$$

$$\psi \, \mathbf{R}_{\leq n} \, \varphi \equiv \varphi \wedge (\psi \vee \bigwedge_{i=1}^{n} \widehat{\circ}_i(\psi \, \mathbf{R}_{\leq (n-i)} \, \varphi)) \tag{12}$$

The same applies for the dual past operators. □

Finally, the next theorem contains a pair of equivalences that, when dealing with finite intervals, can be used to recursively unfold *until* and *release* into combinations of *next* with Boolean operators (an analogous result applies for *since*, *trigger* and *previous* due to temporal duality).

Theorem 2 (Next-unfolding). *For metric formulas ψ and φ and for $m, n \in \mathbb{N}$ such that $0 < m$ and $m < n-1$ we have:*

$$\psi \, \mathbf{U}_{[m..n)} \, \varphi \equiv \bigvee_{i=1}^{m} \circ_i(\psi \, \mathbf{U}_{[m-i..n-i)} \, \varphi) \vee \bigvee_{i=m+1}^{n-1} \circ_i(\psi \, \mathbf{U}_{\leq (n-1-i)} \, \varphi) \tag{13}$$

$$\psi \, \mathbf{R}_{[m..n)} \, \varphi \equiv \bigwedge_{i=1}^{m} \widehat{\circ}_i(\psi \, \mathbf{R}_{[(m-i)..(n-i))} \, \varphi) \wedge \bigwedge_{i=m+1}^{n-1} \widehat{\circ}_i(\psi \, \mathbf{R}_{\leq (n-1-i)} \, \varphi) \tag{14}$$

The same applies for the dual past operators. □

As an example, consider the metric formula $p \,\mathbf{U}_{[2..4)}\, q$.

$$\begin{aligned} p \,\mathbf{U}_{[2..4)}\, q &\equiv \bigvee_{i=1}^{2} \circ_i (p \,\mathbf{U}_{[(2-i)..(4-i))}\, q) \vee \bigvee_{i=2+1}^{3} \circ_i (p \,\mathbf{U}_{\leq(3-i)}\, q) \\ &\equiv \circ_1 (p \,\mathbf{U}_{[1..3)}\, q) \vee \circ_2 (p \,\mathbf{U}_{\leq 1}\, q) \vee \circ_3 (p \,\mathbf{U}_0\, q) \\ &\equiv \circ_1 (p \,\mathbf{U}_{[1..3)}\, q) \vee \circ_2 (q \vee (p \wedge \circ_1 q)) \vee \circ_3 q \\ &\equiv \circ_1 (\circ_1 (q \vee (p \wedge \circ_1 q)) \vee \circ_2 q) \vee \circ_2 (q \vee (p \wedge \circ_1 q)) \vee \circ_3 q \end{aligned}$$

Another useful result that can be applied to unfold metric operators is the following range splitting theorem.

Theorem 3 (Range splitting). *For metric formulas ψ and φ, we have*

$$\begin{aligned} \psi \,\mathbf{U}_{[m..n)}\, \varphi &\equiv (\psi \,\mathbf{U}_{[m..i)}\, \varphi) \vee (\psi \,\mathbf{U}_{[i..n)}\, \varphi) \qquad \text{for all } i \in [m..n) \\ \psi \,\mathbf{R}_{[m..n)}\, \varphi &\equiv (\psi \,\mathbf{R}_{[m..i)}\, \varphi) \wedge (\psi \,\mathbf{R}_{[i..n)}\, \varphi) \qquad \text{for all } i \in [m..n) \end{aligned}$$

The same applies for the dual past operators. □

3 Translation into Monadic Quantified Here-and-There with Difference Constraints

In a similar spirit as the well-known translation of Kamp [19] from LTL to first-order logic, we consider a translation from MHT into a first-order version of HT, more precisely, a function-free fragment of the logic of Quantified Here-and-There with static domains (QHT^s in [25]). The word *static* means that the first-order domain D is fixed for both worlds, here and there. We refer to our fragment of QHT^s as *monadic QHT with difference constraints* ($QHT[\preccurlyeq_\delta]$). In this logic, the static domain is a subset $D \subseteq \mathbb{N}$ of the natural numbers containing at least the element $0 \in D$. Intuitively, D corresponds to the set of relevant time points (i.e. those associated to states) considered in each model. Note that the first state is always associated with time $0 \in D$.

The syntax of $QHT[\preccurlyeq_\delta]$ is the same as for first-order logic with several restrictions: First, there are no functions other than the 0-ary function (or constant) '0' always interpreted as the domain element 0 (when there is no ambiguity, we drop quotes around constant names). Second, all predicates are monadic except for a family of binary predicates of the form $\preccurlyeq_\delta$ with $\delta \in \mathbb{Z} \cup \{\omega\}$ where δ is understood as part of the predicate name. For simplicity, we write $x \preccurlyeq_\delta y$ instead of $\preccurlyeq_\delta(x, y)$ and $x \preccurlyeq_\delta y \preccurlyeq_{\delta'} z$ to stand for $x \preccurlyeq_\delta y \wedge y \preccurlyeq_{\delta'} z$. Unlike monadic predicates, the interpretation of $x \preccurlyeq_\delta y$ is static (it does not vary in worlds here and there) and intuitively means that the difference $x - y$ in time points is smaller or equal than δ. A first-order formula φ satisfying all these restrictions is called a *first-order metric formula* or *FOM-formula* for short. A formula is a *sentence* if it contains no free variables. For instance, we will see that the metric formula (3) can be equivalently translated into the FOM-sentence:

$$\forall x\, (x \preccurlyeq_0 0 \wedge push(x) \rightarrow \exists y\, (x \preccurlyeq_{-1} y \preccurlyeq_{14} x \wedge \forall z\, (y \preccurlyeq_0 z \preccurlyeq_{30} y \rightarrow green(z)))) \tag{15}$$

We sometimes handle *partially grounded* FOM sentences where some variables in predicate arguments have been directly replaced by elements from D. For instance, if we represent (15) as $\forall x\ \varphi(x)$, the expression $\varphi(4)$ stands for:

$$4 \preccurlyeq_0 0 \wedge push(4) \rightarrow \exists y\,(x \preccurlyeq_{-1} y \preccurlyeq_{14} x \wedge \forall z\,(y \preccurlyeq_0 z \preccurlyeq_{30} y \rightarrow green(z)))$$

and corresponds to a partially grounded FOM-sentence where the domain element 4 is used as predicate argument in atoms $4 \preccurlyeq_0 0$ and $push(4)$.

A $QHT[\preccurlyeq_\delta]$-*signature* is simply a set of monadic predicates $\mathcal{P}$. Given D as above, $Atoms(D, \mathcal{P})$ denotes the set of all ground atoms $p(n)$ for every monadic predicate $p \in \mathcal{P}$ and every $n \in D$. A $QHT[\preccurlyeq_\delta]$-interpretation for signature $\mathcal{P}$ has the form $\langle D, H, T\rangle$ where $D \subseteq \mathbb{N}$, $0 \in D$ and $H \subseteq T \subseteq Atoms(D, \mathcal{P})$.

Definition 4 ($QHT[\preccurlyeq_\delta]$-satisfaction; [25]). *A $QHT[\preccurlyeq_\delta]$-interpretation $\mathcal{M} = \langle D, H, T\rangle$ satisfies a (partially grounded) FOM-sentence φ, written $\mathcal{M} \models \varphi$, if the following conditions hold:*

1. $\mathcal{M} \models \top$ *and* $\mathcal{M} \not\models \bot$
2. $\mathcal{M} \models p(d)$ *iff* $p(d) \in H$
3. $\mathcal{M} \models t_1 \preccurlyeq_\delta t_2$ *iff* $t_1 - t_2 \leq \delta$, *with* $t_1, t_2 \in D$
4. $\mathcal{M} \models \varphi \wedge \psi$ *iff* $\mathcal{M} \models \varphi$ *and* $\mathcal{M} \models \psi$
5. $\mathcal{M} \models \varphi \vee \psi$ *iff* $\mathcal{M} \models \varphi$ *or* $\mathcal{M} \models \psi$
6. $\mathcal{M} \models \varphi \rightarrow \psi$ *iff* $\langle D, X, T\rangle \not\models \varphi$ *or* $\langle D, X, T\rangle \models \psi$, *for* $X \in \{H, T\}$
7. $\mathcal{M} \models \forall x\ \varphi(x)$ *iff* $\mathcal{M} \models \varphi(t)$, *for all* $t \in D$
8. $\mathcal{M} \models \exists x\ \varphi(x)$ *iff* $\mathcal{M} \models \varphi(t)$, *for some* $t \in D$ □

We can read the expression $x \preccurlyeq_\delta y$ as just another way of writing the difference constraint $x - y \leq \delta$. When δ is an integer, we may see it as a lower bound $x - \delta \leq y$ for y or as an upper bound $x \leq y + \delta$ for x. For $\delta = \omega$, $x \preccurlyeq_\omega y$ is equivalent to $\top$ since it amounts to the comparison $x - y \leq \omega$. An important observation is that this difference predicate $\preccurlyeq_\delta$ satisfies the excluded middle axiom, that is, the following formula is a $QHT[\preccurlyeq_\delta]$-tautology:

$$\forall x\,\forall y\,(\ x \preccurlyeq_\delta y \vee \neg(x \preccurlyeq_\delta y)\ ) \tag{16}$$

for every $\delta \in \mathbb{Z} \cup \{\omega\}$. We provide next several useful abbreviations:

$$x \prec_\delta y \stackrel{def}{=} \neg(y \preccurlyeq_{-\delta} x)$$

$$x \leq y \stackrel{def}{=} x \preccurlyeq_0 y \qquad\qquad x \neq y \stackrel{def}{=} \neg(x = y)$$

$$x = y \stackrel{def}{=} (x \leq y) \wedge (y \leq x) \qquad\qquad x < y \stackrel{def}{=} (x \leq y) \wedge (x \neq y)$$

For any pair $\odot$, $\oplus$ of comparison symbols, we extend the abbreviation $x \odot y \oplus z$ to stand for the conjunction $x \odot y \wedge y \oplus z$. Note that the above derived order relation $x \leq y$ captures the one used in Kamp's original translation [19] for LTL.

Equilibrium models for first-order theories are defined as in [25].

Definition 5 (Quantified Equilibrium Model; [25]). *Let φ be a first-order formula. A total $QHT[\preccurlyeq_\delta]$-interpretation $\langle D, T, T\rangle$ is a first-order equilibrium model of φ if $\langle D, T, T\rangle \models \varphi$ and there is no $H \subset T$ satisfying $\langle D, H, T\rangle \models \varphi$.* □

Before presenting our translation, we need to remark that we consider non-empty intervals of the form $[m..n)$ with $m < n$.

Definition 6 (First-order encoding). *Let φ be a metric formula over $\mathcal{A}$. We define the translation $[\varphi]_x$ of φ for some time point $x \in \mathbb{N}$ as follows:*

$$
\begin{aligned}
[\bot]_x &\stackrel{def}{=} \bot \\
[p]_x &\stackrel{def}{=} p(x), \quad \textit{for any } p \in \mathcal{A} \\
[\varphi \otimes \psi]_x &\stackrel{def}{=} [\varphi]_x \otimes [\beta]_x \ , \quad \textit{for any connective } \otimes \in \{\wedge, \vee, \to\} \\
[\circ_{[m,n)} \psi]_x &\stackrel{def}{=} \exists y \, (x < y \wedge (\neg \exists z \; x < z < y) \wedge x \preccurlyeq_{-m} y \prec_n x \wedge [\psi]_y) \\
[\widehat{\circ}_{[m,n)} \psi]_x &\stackrel{def}{=} \forall y \, (x < y \wedge (\neg \exists z \; x < z < y) \wedge x \preccurlyeq_{-m} y \prec_n x \to [\psi]_y) \\
[\varphi \, \mathbf{U}_{[m,n)} \, \psi]_x &\stackrel{def}{=} \exists y \ (x \leq y \wedge x \preccurlyeq_{-m} y \prec_n x \wedge [\psi]_y \wedge \forall z \, (x \leq z < y \to [\varphi]_z)) \\
[\varphi \, \mathbf{R}_{[m,n)} \, \psi]_x &\stackrel{def}{=} \forall y \ ((x \leq y \wedge x \preccurlyeq_{-m} y \prec_n x) \to ([\psi]_y \vee \exists z \, (x \leq z < y \wedge [\varphi]_z))) \\
[\bullet_{[m,n)} \psi]_x &\stackrel{def}{=} \exists y \, (y < x \wedge \neg \exists z \ (y < z < x) \wedge x \prec_n y \preccurlyeq_{-m} x \wedge [\psi]_y) \\
[\widehat{\bullet}_{[m,n)} \psi]_x &\stackrel{def}{=} \forall y \, ((y < x \wedge \neg \exists z \ (y < z < x) \wedge x \prec_n y \preccurlyeq_{-m} x) \to [\psi]_y) \\
[\varphi \, \mathbf{S}_{[m,n)} \, \psi]_x &\stackrel{def}{=} \exists y \ (y \leq x \wedge x \prec_n y \preccurlyeq_{-m} x \wedge [\psi]_y \wedge \forall \, (y < z \leq x \to [\varphi]_z)) \\
[\varphi \, \mathbf{T}_{[m,n)} \, \psi]_x &\stackrel{def}{=} \forall y \ ((y \leq x \wedge x \prec_n y \preccurlyeq_{-m} x) \to ([\psi]_y \vee \exists z \, (y < z \leq x \wedge [\varphi]_z)))
\end{aligned}
$$

□

Each quantification introduces a new variable. For instance, consider the translation of (3) at point $x = 0$. Let us denote (3) as $\square(\mathit{push} \to \alpha)$ where $\alpha := \Diamond_{[1..15)}(\square_{\leq 30}\, \mathit{green})$. Then, if we translate the outermost operator $\square$, we get:

$$
\begin{aligned}
&[\square(\mathit{push} \to \alpha)]_0 \\
&= [\bot \ \ \mathbf{R}_{[0..\omega)} \ (\mathit{push} \to \alpha)]_0 \\
&= \forall y \ \ ((0 \leq y \wedge 0 \preccurlyeq_{-0} y \prec_\omega 0) \to ([\mathit{push} \to \alpha]_y \vee \exists z \, (0 \leq z < y \wedge \bot))) \\
&\equiv \forall y \ (0 \leq y \wedge 0 \leq y \wedge \top \to ([\mathit{push}]_y \to [\alpha]_y) \vee \bot) \\
&\equiv \forall y \ (0 \leq y \wedge \mathit{push}(y) \to [\alpha]_y) \\
&\equiv \forall x \ (0 \leq x \wedge \mathit{push}(x) \to [\alpha]_x)
\end{aligned}
$$

where we renamed the quantified variable for convenience. If we proceed further, with α as $\Diamond_{[1..15)}\beta$ letting $\beta := (\square_{\leq 30}\, \mathit{green})$, we obtain:

$$
\begin{aligned}
[\alpha]_x &= [\Diamond_{[1..15)}\beta]_x \\
&= [\top \ \ \mathbf{U}_{[1..15)} \ \ \beta]_x \\
&= \exists y \ \ (x \leq y \wedge x \preccurlyeq_{-1} y \prec_{15} x \wedge [\beta]_y \wedge \forall z \, (x \leq z < y \to \top)) \\
&\equiv \exists y \ \ (x \preccurlyeq_{-1} y \prec_{15} x \wedge [\beta]_y) \equiv \exists y \ \ (x \preccurlyeq_{-1} y \preccurlyeq_{14} x \wedge [\beta]_y)
\end{aligned}
$$

Finally, the translation of β at y amounts to:

$$\begin{aligned}
&[\Box_{\leq 30}\ green]_y \\
&= [\bot\ \mathbf{R}_{[0..30)}\ \ green]_y \\
&= \forall y'\ (\ y \leq y' \wedge y \preccurlyeq_{-0} y' \prec_{30} y \rightarrow green(y') \vee \exists z\,(y \leq z < y' \wedge \bot)\ ) \\
&\equiv \forall y'\ (\ y \leq y' \wedge y \preccurlyeq_{0} y' \wedge y' \prec_{30} y \rightarrow green(y')\ ) \\
&\equiv \forall y'\ (\ y \preccurlyeq_{0} y' \prec_{30} y \rightarrow green(y')\ ) \\
&\equiv \forall z\ (\ y \preccurlyeq_{0} z \prec_{30} y \rightarrow green(z)\ )
\end{aligned}$$

so that, when joining all steps together, we get the formula (15) given above.

The following model correspondence between MHT_f and $QHT[\preccurlyeq_\delta]$ interpretations can be established. Given a timed trace $(\langle \mathbf{H}, \mathbf{T}\rangle, \tau)$ of length $\lambda > 0$ for signature $\mathcal{A}$, we define the first-order signature $\mathcal{P} = \{p/1 \mid p \in \mathcal{A}\}$ and a corresponding $QHT[\preccurlyeq_\delta]$ interpretation $\langle D, H, T\rangle$ where $D = \{\tau(i) \mid i \in [0..\lambda)\}$, $H = \{p(\tau(i)) \mid i \in [0..\lambda)$ and $p \in H_i\}$ and $T = \{p(\tau(i)) \mid i \in [0..\lambda)$ and $p \in T_i\}$. Under the assumption of strict semantics, the following model correspondence can be proved by structural induction.

Theorem 4. *Let φ be a metric temporal formula, $(\langle \mathbf{H}, \mathbf{T}\rangle, \tau)$ a metric trace, $\langle D, H, T\rangle$ its corresponding $QHT[\preccurlyeq_\delta]$ interpretation and $i \in [0..\lambda)$.*

$$(\langle \mathbf{H}, \mathbf{T}\rangle, \tau), i \models \varphi \ \ \textit{iff}\ \ \langle D, H, T\rangle \models [\varphi]_{\tau(i)} \tag{17}$$

$$(\langle \mathbf{T}, \mathbf{T}\rangle, \tau), i \models \varphi \ \ \textit{iff}\ \ \langle D, T, T\rangle \models [\varphi]_{\tau(i)} \tag{18}$$

□

4 Discussion

Seen from far, we have presented an extension of the logic of Here-and-There with qualitative and quantitative temporal constraints. More closely, our logics MHT and MEL can be seen es metric extensions of the linear-time logics THT and TEL obtained by constraining temporal operators by intervals over natural numbers. The current approach generalizes the previous metric extension of TEL from [9] by uncoupling the ordinal position i of a state in the trace from its location in the time line $\tau(i)$, which indicates now the elapsed time since the beginning of that trace. Thus, while $\Diamond_{[5..5]}\ p$ meant in [9] that p must hold exactly after 5 transitions, it means here that there must be some future state (after $n > 0$ transitions) satisfying p and located 5 time units later. As a first approach, we have considered time points as natural numbers, $\tau(i) \in \mathbb{N}$. Our choice of a discrete rather than continuous time domain is primarily motivated by our practical objective to implement the logic programming fragment of MEL on top of existing temporal ASP systems, like *telingo*, and thus to avoid undecidability.

The need for quantitative time constraints is well recognized and many metric extensions have been proposed. For instance, actions with durations are considered in [27] in an action language adapting a state-based approach.

Interestingly, quantitative time constraints also gave rise to combining ASP with Constraint Solving [3]; this connection is now semantically reinforced by our translation advocating the enrichment of ASP with difference constraints. Even earlier, metric extensions of Logic Programming were proposed in [6]. As well, metric extensions of Datalog are introduced in [28] and applied to stream reasoning in [29]. An ASP-based approach to stream reasoning is elaborated in abundance in [4]. Streams can be seen as infinite traces. Hence, apart from certain dedicated concepts, like time windows, such approaches bear a close relation to metric reasoning. Detailing this relationship is an interesting topic of future research. More remotely, metric constructs were used in trace alignment [13], scheduling [22], and an extension to Golog [18].

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