Temporal Equilibrium Logic: a first approach*

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Abstract. In this paper we introduce an extension of Equilibrium Logic (a logical characterisation of the Answer Set Semantics for logic programs) consisting in the inclusion of modal temporal operators, as those used in Linear Temporal Logic. As a result, we obtain a very expressive formalism that allows nonmonotonic reasoning for temporal domains. To show an example of its utility, we present a translation of a language for reasoning about actions into this formalism.

1 Introduction

One of the areas of Logic Programming that has attracted more research interest during the last decade is *Answer Set Programming* (ASP). The origins of this area come from the achievement of a suitable semantics (first called *stable models* [1] and later generalized into *answer sets* [2]) for interpreting default negation in *normal* logic programs[1]. The availability of this semantics allowed considering logic programs as a general knowledge representation formalism, especially for solving default and non-monotonic reasoning problems. Under this new focusing, a process of gradual syntactic extensions followed, most of them eventually captured by [3], a completely logical characterisation of answer sets into the non-monotonic formalism of *Equilibrium Logic*. This approach offers a standard logical interpretation for any arbitrary theory, removing any syntactic restriction while generalising all previous extensions of answer set semantics.

Apart from with this nice logical characterisation, the recent success of ASP is mostly due to several existing tools (see [4]) offering a high efficiency degree for computing answer sets of a normal logic program (an NP-complete problem). The price to be paid for this efficiency is the impossibility of handling function symbols, restriction that guarantees a finite domain and allows grounding the program into a finite set of propositional rules. *Planning* for transition systems constitutes one of the most popular application areas of ASP. Typically, the logic program is used to encode the initial state, the transition rules and the goal description. Default negation plays here a crucial role, as it allows avoiding the frame problem [5] by representing the default rule of *inertia*: "if a fluent (or

* This research is partially supported by Spanish Ministry MEC, research project TIN-15455-C03-02.

[1] Those with rules containing an atom in the head and a body consisting of a conjunction of literals.

temporal variable) is not known to have changed, it will remain unchanged by default." The general planning problem lies in a harder class (PSPACE-complete) than the one covered by ASP solvers. This means in practice that, in order to solve planning problems, it is first required to *fix the length* l of the plan to be obtained. Usually, the plan search consists in gradually incrementing this length parameter $l = 0, 1, 2, 3, \ldots$ until a plan is obtained. A first obvious drawback of this approach is that it is not possible to establish when a given planning problem has *no solution* of any length at all. A second and more elaborated problem is that it is impossible to establish when two descriptions of the same transition system are *strongly equivalent*, i.e., when they will behave in the same way regardless any additional rules we include and any path length we consider.

In the paper we consider an extension of Equilibrium Logic (and so, of logic programming) to deal with modal operators as those handled in Linear Temporal Logic (LTL) [6] [7]: $\Box$ (forever), $\Diamond$ (eventually), $\bigcirc$ (next), $\mathcal{U}$ (until) and $\mathcal{W}$ (waiting for). Equilibrium Logic is defined as a model minimisation on top of a monotonic framework, the non-classical intermediate logic of *here-and-there* (HT) [8]. The temporal extension will be defined in a similar way, introducing first the temporal extension of HT (called THT), and then a minimisation criterion. For space reasons, we omit the description of Equilibrium logic, as it can be defined as a particular case of its temporal extension.

The paper is organised as follows. Section 2 describes the semantics of THT and Section 3 defines a models minimisation criterion for THT models that leads to Temporal Equilibrium Logic (TEL). In the next section, we show the utility of TEL providing the translation of a language for reasoning about actions. Finally, Section 5 contains the conclusions and future work.

2 Linear Temporal Here-and-There (THT)

The logic of *Linear Temporal Here-and-There* (THT) is defined as follows. We start from a finite set of atoms Σ called the *propositional signature.* A (temporal) *formula* is defined as any combination of the classical connectives $\wedge, \vee, \rightarrow, \bot$ with the temporal operators $\Box, \Diamond, \bigcirc$ and the atoms in Σ. Operators, $\Box$, $\Diamond$ and $\bigcirc$ are unary, whereas $\mathcal{U}$ and $\mathcal{W}$ are binary. A formula is said to be *non-modal* if it does not contain any of these temporal operators. Negation is defined as $\neg\varphi \stackrel{def}{=} \varphi \rightarrow \bot$. As usual, $\varphi \leftrightarrow \psi$ stands for $(\varphi \rightarrow \psi) \wedge (\psi \rightarrow \varphi)$. We also allow the abbreviation $\bigcirc^i \stackrel{def}{=} \bigcirc(\bigcirc^{i-1}\varphi)$ for $i > 0$ and $\bigcirc^0\varphi \stackrel{def}{=} \varphi$.

A (temporal) *interpretation* M is a possibly infinite sequence of pairs $\langle H_i, T_i\rangle$ with $i = 0, 1, 2, \ldots$ where $H_i \subseteq T_i$ are sets of atoms. For simplicity, given a temporal interpretation, we write $\overline{H}$ (resp. $\overline{T}$) to denote the sequence of pair components $H_0, H_1, \ldots$ (resp. $T_0, T_1, \ldots$). Using this notation, we will sometimes abbreviate the interpretation as $M = \langle\overline{H}, \overline{T}\rangle$. An interpretation $M = \langle\overline{H}, \overline{T}\rangle$ is said to be total when $\overline{H} = \overline{T}$. We say that an interpretation $M = \langle\overline{H}, \overline{T}\rangle$ satisfies a formula φ at some sequence index i, written $M, i \models \varphi$, when any of the following hold:

1. $M, i \models p$ if $p \in H_i$, for any atom p.
2. $M, i \models \varphi \wedge \psi$ if $M, i \models \varphi$ and $M, i \models \psi$.
3. $M, i \models \varphi \vee \psi$ if $M, i \models \varphi$ or $M, i \models \psi$.
4. $\langle \overline{H}, \overline{T} \rangle, i \models \varphi \rightarrow \psi$ if $\langle X, \overline{T} \rangle, i \neg \models \varphi$ or $\langle X, \overline{T} \rangle \models \psi$ for all $X \in \{\overline{H}, \overline{T}\}$.
5. $M, i \models \bigcirc\varphi$ if i is the greatest subindex in M or $M, i{+}1 \models \varphi$ otherwise.
6. $M, i \models \Box\varphi$ if for all $j \geq i$, $M, j \models \varphi$.
7. $M, i \models \Diamond\varphi$ if there exists some $j \geq i$, $M, j \models \varphi$.
8. $\varphi \, \mathcal{U} \, \psi$ if $\exists j \geq i, \quad M, j \models \psi$ and $\forall k$ s.t. $i \leq k < j, \quad M, k \models \varphi$
9. $\varphi \, \mathcal{W} \, \psi$ if either $M, i \models \varphi \, \mathcal{U} \, \psi$ or, for all $j \geq i, \;\; M, j \models \varphi$.

Notice that the semantics for $\bigcirc\varphi$ leaves free the possibility of using both finite and infinite sequences. If the sequence is finite, at the last state $\bigcirc\varphi$ will be true regardless φ. To fix the final state, thus, we can define the derived formula $\mathbf{end} \stackrel{def}{=} \bigcirc\bot$. Finite (resp. infinite) narratives can then be forced by including the axiom $\Diamond\mathbf{end}$ (resp. $\neg\Diamond\mathbf{end}$).

The logic of THT is an orthogonal combination of the logic of HT and the (standard) linear temporal logic (LTL) [7]. When we restrict temporal interpretations to finite sequences of length 1, that is $\langle H_0, T_0 \rangle$ and disregard temporal operators, we obtain the logic of HT. On the other hand, if we restrict the semantics to total interpretations, $\langle \overline{T}, \overline{T} \rangle, i \models \varphi$ corresponds to satisfaction of formulas $\overline{T}, i \models \varphi$ in LTL.

A *theory* is any set of formulas. An interpretation M is a *model* of a theory Γ, written $M \models \Gamma$, if $M, 0 \models \alpha$, for all formula $\alpha \in \Gamma$. A formula φ is *valid* if $M, 0 \models \varphi$ for any M. The following are valid formulas in THT (and in LTL too):

$$\neg\Box\varphi \leftrightarrow \Diamond\neg\varphi \tag{1}$$
$$\neg\Diamond\varphi \leftrightarrow \Box\neg\varphi \tag{2}$$
$$\bigcirc\neg\varphi \leftrightarrow \neg \bigcirc \varphi \tag{3}$$
$$\bigcirc(\varphi \wedge \psi) \leftrightarrow \bigcirc\varphi \wedge \bigcirc\psi \tag{4}$$
$$\bigcirc(\varphi \vee \psi) \leftrightarrow \bigcirc\varphi \vee \bigcirc\psi \tag{5}$$
$$\bigcirc(\varphi \rightarrow \psi) \leftrightarrow (\bigcirc\varphi \rightarrow \bigcirc\psi) \tag{6}$$
$$\varphi \, \mathcal{U} \, \psi \leftrightarrow \varphi \, \mathcal{W} \, \psi \wedge \Diamond\psi \tag{7}$$
$$\varphi \, \mathcal{W} \, \psi \leftrightarrow \varphi \, \mathcal{U} \, \psi \vee \Box\varphi \tag{8}$$

Note that (3)-(6) allow shifting $\bigcirc$ through any combination of non-modal operators. In this way, any theory not containing $\Box, \Diamond, \mathcal{U}, \mathcal{W}$ can be transformed into a combination of non-modal connectives applied to expressions like $\bigcirc^i p$, where p is an atom. Notice also that, even though (1) and (2) are valid, we cannot further define $\Box$ in terms of $\Diamond$ or vice versa, as happens in LTL. For instance, as a countermodel of $\Box p \leftrightarrow \neg\Diamond\neg p$ in THT it suffices to take an interpretation $\langle \overline{H}, \overline{T} \rangle$ with all $T_i = \{p\}$ but some $H_i = \emptyset$.

We can alternatively represent any interpretation $M = \langle \overline{H}, \overline{T} \rangle$ as a possibly infinite sequence of mappings V_i assigning a truth value to each atom $V_i : \Sigma \rightarrow \{0, 1, 2\}$ where 0 stands for false, 2 for true and 1 for undefined. We can define a valuation for any formula φ at any sequence index i, written $M_i(\varphi)$, by just considering which formulas are satisfied by $\langle \overline{H}, \overline{T} \rangle$ (which will be assigned 2), not satisfied by $\langle \overline{T}, \overline{T} \rangle$ (which will be assigned 0) or none of the two (which will be 1). Thus, the definitions in the previous section allow deriving the following conditions:

1. $M_i(p) \stackrel{def}{=} V_i(p)$
2. $M_i(\varphi \wedge \psi) \stackrel{def}{=} min(M_i(\varphi), M_i(\psi))$
3. $M_i(\varphi \vee \psi) \stackrel{def}{=} max(M_i(\varphi), M_i(\psi))$
4. $M_i(\varphi \to \psi) \stackrel{def}{=} \begin{cases} 2 & \text{if } M_i(\varphi) \leq M_i(\psi) \\ M(\psi) & \text{otherwise} \end{cases}$
5. $M_i(\bigcirc \varphi) \stackrel{def}{=} \begin{cases} 2 & \text{if } i \text{ is the greatest subindex in } M \\ M_{i+1}(\varphi) & \text{otherwise} \end{cases}$
6. $M_i(\Box \varphi) \stackrel{def}{=} min\{M_j(\varphi) \mid j \geq i\}$
7. $M_i(\Diamond \varphi) \stackrel{def}{=} max\{M_j(\varphi) \mid j \geq i\}$
8. $M_i(\varphi\ \mathcal{U}\ \psi) \stackrel{def}{=} \begin{cases} 2 \text{ if exists } j \geq i, M_j(\varphi) = 2 \\ \quad \text{and } M_k(\psi) = 2 \text{ for all } k \in [i, j-1] \\ 0 \text{ if for all } j \geq i : M_j(\varphi) = 0 \\ \quad \text{or } M_k(\psi) = 0 \text{ for some } k \in [i, j-1] \\ 1 \text{ otherwise} \end{cases}$
9. $M_i(\varphi\ \mathcal{W}\ \psi) \stackrel{def}{=} M_i(\varphi\ \mathcal{U}\ \psi \vee \Box \varphi)$

Using this alternative definition, an interpretation M satisfies a formula φ when $M_0(\varphi) = 2$.

3 Linear Temporal Equilibrium Logic (TEL)

We can now proceed to describe the models selection criterion that defines temporal equilibrium models. Given two interpretations $M = \langle \overline{H}, \overline{T} \rangle$ and $M' = \langle \overline{H}', \overline{T}' \rangle$ we say that M is *lower than* M', written $M \leq M'$, when $\overline{T} = \overline{T}'$ and for all $i \geq 0$, $H_i \subseteq H'_i$. As usual, $M < M'$ stands for $M \leq M'$ but $M \neq M'$.

Definition 1 (Temporal Equilibrium Model). *An interpretation M is a* temporal equilibrium model *of a theory Γ if M is a total model of Γ and there is no other $M' < M$, $M', 0 \models \Gamma$.* □

Note that any temporal equilibrium model is total, that is, it has the form $\langle \overline{T}, \overline{T} \rangle$ and so can be actually seen as an interpretation $\overline{T}$ in the standard LTL.

Example 1. The temporal equilibrium models of theory $\{\Diamond p\}$ correspond to LTL interpretations like $\left[\ \emptyset\ \right]^* \{p\} \left[\ \emptyset\ \right]^*$.

Proof. Take any a total interpretation $M = \langle \overline{T}, \overline{T} \rangle$ with some $T_i = \{p\}$. Clearly, $M, 0 \models \Diamond p$, but its minimality will depend on whether there exists another $j \neq i$ with $T_j = \{p\}$ in $\overline{T}$. Assume there exists such a j: then it is easy to see that the interpretation $\{\overline{H}, \overline{T}\}$ with $H_j = \emptyset$ and $\overline{H}_k = \overline{T}_k$ for $k \neq j$ is also a model of $\Diamond p$ (since $H_i = T_i = \{p\}$) whereas it is strictly lower, so it would not be a temporal equilibrium model. Assume, on the contrary, that T_i is the unique state containing p, so $T_k = \emptyset$ for all $k \neq i$. Then, the only possible smaller interpretation would be $\{\overline{H}, \overline{T}\}$ with $H_k = T_k = \emptyset$ for $k \neq i$ and $H_i = \emptyset$, but this is not a THT model of $\Diamond p$, as p would not occur in $\overline{H}$. □

Example 2. Consider the theory Γ just consisting of the formula $\Box(\neg p \rightarrow \bigcirc p)$. Its temporal equilibrium models correspond to LTL interpretations like $\emptyset \left[\{p\} \ \emptyset \right]^*$ or $\emptyset \left[\{p\} \ \emptyset \right]^* \{p\}$.

Proof. For an informal sketch, note that any solution with p true at T_0 will not be minimal. Once $\neg p$ is fixed at the initial state, we must get p at T_1 to satisfy the formula. Then p true at T_2 would not be minimal, so it must be false, and so on. □

Proposition 1. *Let Γ be a combination of non-modal connectives $\wedge, \vee, \neg, \rightarrow, \bot$ with expressions like $\bigcirc^i p$, being p an atom, and let n be the maximum value for i in all $\bigcirc^i p$ occurring in Γ. Then $\langle \overline{T}, \overline{T} \rangle$ is a temporal equilibrium model of Γ iff (1) $T_i = \emptyset$ for all $i > n$; and (2) $\langle X, X \rangle$ with $X = \bigcup_{i=0}^{n} \{\bigcirc^i p \mid p \in T_i\}$ is an equilibrium model of Γ, reading each '$\bigcirc^i p$' as a new atom in the signature.* □

That is, once $\Box, \Diamond, \mathcal{U}$ and $\mathcal{W}$ are removed, we can reduce TEL to (non-temporal) Equilibrium Logic for an extended signature with atoms like $\bigcirc^i p$.

4 Translation of an action description language

In this section we briefly introduce an example of how TEL can be naturally used to translate action description languages. To this aim, we consider an encoding of language $\mathcal{B}$ [9], obtaining a similar translation to the one previously presented in [10] for a different temporal nonmonotonic formalism. Language $\mathcal{B}$ defines a two-sorted signature: the set $\mathcal{A}$ of action names and the set $\mathcal{F}$ of fluent names. By *fluent literal* we understand any fluent name F or its explicit negation, we denote[2] as $\overline{F}$. A *state* is a maximal consistent (that is, not containing both F and $\overline{F}$) set of fluent literals. An action description $D = Y \cup Z$ consists of a set Y of *dynamic laws* of the form (A **causes** L **if** φ) and a set Z of *static laws* of the form (L **if** φ) where A is an action, L a fluent literal and φ a conjunction of fluent literals. The semantics of $\mathcal{B}$ is defined as follows. A set s of literals is *closed under* Z iff for each (L **if** φ) $\in Z$ such that s satisfies φ, $L \in s$. Let $Cn_Z(s)$ be the smallest set of literals containing s that is closed under Z. For a given action description $D = Y \cup Z$, we define a transition system as a triple $\langle S, V, R \rangle$ where:

1. S is the set of all states that are closed under Z.
2. $V(P, s) = s(P)$ is a valuation function that assigns a truth value (*true* or *false*), to each fluent P at each state $s \in S$.

[2] We use this notation to avoid confusion with default negation, represented as $\neg$ in this paper.

3. R is the transition relation containing all the triples $\langle s, A, s' \rangle$ such that $s' = Cn_Z(E(A,s) \cup (s \cap s'))$ being
$E(A,s) \stackrel{def}{=} \{L : (A \textbf{ causes } L \textbf{ if } \varphi) \in Y$ and s satisfies φ

The translation into TEL is defined as follows. For each static or dynamic law $\phi \in D$ we define $\Pi(\phi)$ as:

$$\Pi(L \textbf{ if } \varphi) \stackrel{def}{=} \Box(\varphi \rightarrow L) \qquad \Pi(A \textbf{ causes } L \textbf{ if } \varphi) \stackrel{def}{=} \Box(A \wedge \varphi \rightarrow \bigcirc L)$$

The TEL theory $\Pi(D)$ is defined as $\{\Pi(\phi) : \phi \in D\}$ plus the *inertia law*:

$$\Box(F \wedge \neg \bigcirc \overline{F} \rightarrow \bigcirc F) \tag{9}$$
$$\Box(\overline{F} \wedge \neg \bigcirc F \rightarrow \bigcirc \overline{F}) \tag{10}$$

for any fluent F and the axiom schemata:

$$\Diamond \textbf{end} \tag{11}$$
$$F \vee \overline{F} \tag{12}$$
$$\Box(F \wedge \overline{F} \rightarrow \bot) \tag{13}$$
$$\Box(A \vee \neg A) \tag{14}$$
$$\Box(A_i \wedge A_j \rightarrow \bot) \tag{15}$$
$$\Box(\textbf{end} \wedge A \rightarrow \bot) \tag{16}$$

for each $P \in \mathcal{F}$, and $A, A_i, A_j \in \mathcal{A}$, with $A_i \neq A_j$. As explained before, $\Diamond$**end** fixes models with finite length. Axiom (12) completes any fluent value at the initial situation by making it either true F or explicitly false $\overline{F}$. Axiom (13) rejects solutions where both F and $\overline{F}$ hold at the same state[3]. Axiom (14) gives freedom to choose any action occurrence at any state. This is then constrained by axiom (15), which avoids concurrence of two different actions, and axiom (16) that avoids action executions at the last situation in the narrative.

Given a $\mathcal{B}$ transition $\langle s, A, s' \rangle$ we say that a THT interpretation like $\langle \overline{T}, \overline{T} \rangle$ with $\overline{T} = \{T_0, T_1\}$, $T_0 = \{A\} \cup s$ and $T_1 = s'$ *corresponds* to $\langle s, A, s' \rangle$.

Theorem 1. *Given an action description D in $\mathcal{B}$, the set of transitions of D corresponds to the temporal equilibrium models of $\Pi(D)$ plus the axiom* $\neg$**end** $\wedge$ $\bigcirc$**end**, *which fixes finite narratives of length 2.*

Proof. When the narrative length is 2, we may instantiate $\Pi(D)$ unfolding $\Box\phi$ formulas as $\phi \wedge \bigcirc\phi$ and then shifting $\bigcirc$ using (3)-(6) to eventually obtain the

[3] Note that explicit negation has no logical entity in our current definition of TEL, and $\overline{F}$ is just interpreted as one more regular atom.

theory Π':

$$\begin{aligned}
\Pi(L \textbf{ if } \varphi) &= \{\varphi \to L, & (17)\\
&\quad \bigcirc\varphi \to \bigcirc L\} & (18)\\
\Pi(A \textbf{ causes } L \textbf{ if } \varphi) &= \varphi \wedge A \to \bigcirc L & (19)\\
F \wedge \neg \bigcirc \overline{F} &\to \bigcirc F & (20)\\
\overline{F} \wedge \neg \bigcirc F &\to \bigcirc \overline{F} & (21)\\
&F \vee \overline{F} & (22)\\
F \wedge \overline{F} &\to \bot & (23)\\
\bigcirc F \wedge \bigcirc \overline{F} &\to \bot & (24)\\
&A \vee \neg A & (25)\\
&\bigcirc A \vee \neg \bigcirc A & (26)\\
A_i \wedge A_j &\to \bot & (27)\\
\bigcirc A_i \wedge \bigcirc A_j &\to \bot & (28)\\
(\textbf{end} \wedge A \to \bot) \wedge & & (29)\\
\bigcirc \textbf{end} \wedge \bigcirc A &\to \bot & (30)
\end{aligned}$$

As the only modal operator of Π' is $\bigcirc$, we can then apply Proposition 1 to interpret this theory under Equilibrium Logic for signature $\Sigma \cup \{`\bigcirc p' \mid p \in \Sigma\}$ or, what is the same, consider the stable models of Π' under this signature. We can apply the splitting theorem [11] to divide the program Π' into Π'_1 with rules not containing $\bigcirc$, that is (17),(22),(23),(25),(27),(29) and Π'_2 with all the rest. It is not difficult to see that the set of stable models of Π' corresponds to $\{s \cup \{A\} \mid A \in \mathcal{A}$ and s is a complete state closed under $Z, A \in \mathcal{A}\}$, that is, all possible pairs $\langle s, A\rangle$. Now, consider any stable model T of Π' so that $\langle s, A\rangle$ is stable model of Π'_1, whereas $s' = \{p \mid \bigcirc p \in T\}$. Once s and A are fixed, we can simplify Π'_2 so that rules (19) will amount to facts $\bigcirc L$ per each $L \in E(A, s)$, so $E(A, s) \subseteq s'$. On the other hand, if $F \in (s \cap s')$, we get $\bigcirc F \in T$ what together with (24) implies $\neg \bigcirc \overline{F}$, and so the antecedent of (20) holds, and we can replace this rule by the fact $\bigcirc F$. A similar reasoning can be done when $\overline{F} \in (s \cap s')$ to conclude that (21) can be replaced by fact $\bigcirc \overline{F}$. Thus, for a given $\langle s, A, s'\rangle$ we have a program containing a fact $\bigcirc L$ for any $L \in E(A, s) \cup (s \cap s')$. Since static laws Z are also translated for situation 1 by formulas (18) it follows that $s' = Cn_Z(E(A, s) \cup (s \cap s'))$. □

Theorem 2. *Given an action description D in $\mathcal{B}$, any temporal answer set of $\Pi(D)$ corresponds to a history of D.*

Proof. By induction on the length of the finite narrative n, taking as the base case Theorem 1 and generalizing the program instantiation $\Pi(D)^*$ for any $i > 2$.

5 Conclusions

In this paper we have provided a first definition for the linear temporal extension of Equilibrium Logic and gave some examples of its potential utility. As a result, we are able to capture the temporal component of planning problems inside a uniform logical formalism, being able to check properties like the non-existence of plan or the strong equivalence of different representations for the same transition system. On the other hand, the resulting logic may also offer interesting features for the typical application field of LTL, namely formal verification, as its nonmonotonicity allows expressing defaults, as for instance, the previously mentioned rule of inertia.

Much work remains to be done yet, both from a theoretical and a practical point of view. From the former, for instance, we can mention the axiomatisation for THT or a complexity assessment both for THT and TEL entailment. Another important issue is decidability, which seems easy to guess but has not been proved yet. From the practical point of view, obtaining inference methods (like semantic tableaux or temporal extensions of logic programming algorithms) or translating other planning and action languages for their comparison are the two main lines for ongoing work.

Acknowledgements We wish to thank David Pearce, Agustín Valverde, Manuel Ojeda and Alfredo Burrieza for an initial discussion motivating this work.

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