REASONING WITH INACCURATE INFORMATION

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REASONING WITH INACCURATE INFORMATION

It may happen that the information available is incomplete.

In many cases the information available is not sufficient to make a categorical decision.

Sometimes the available information we handle is not exactly true.

The information we use is usually imprecise.

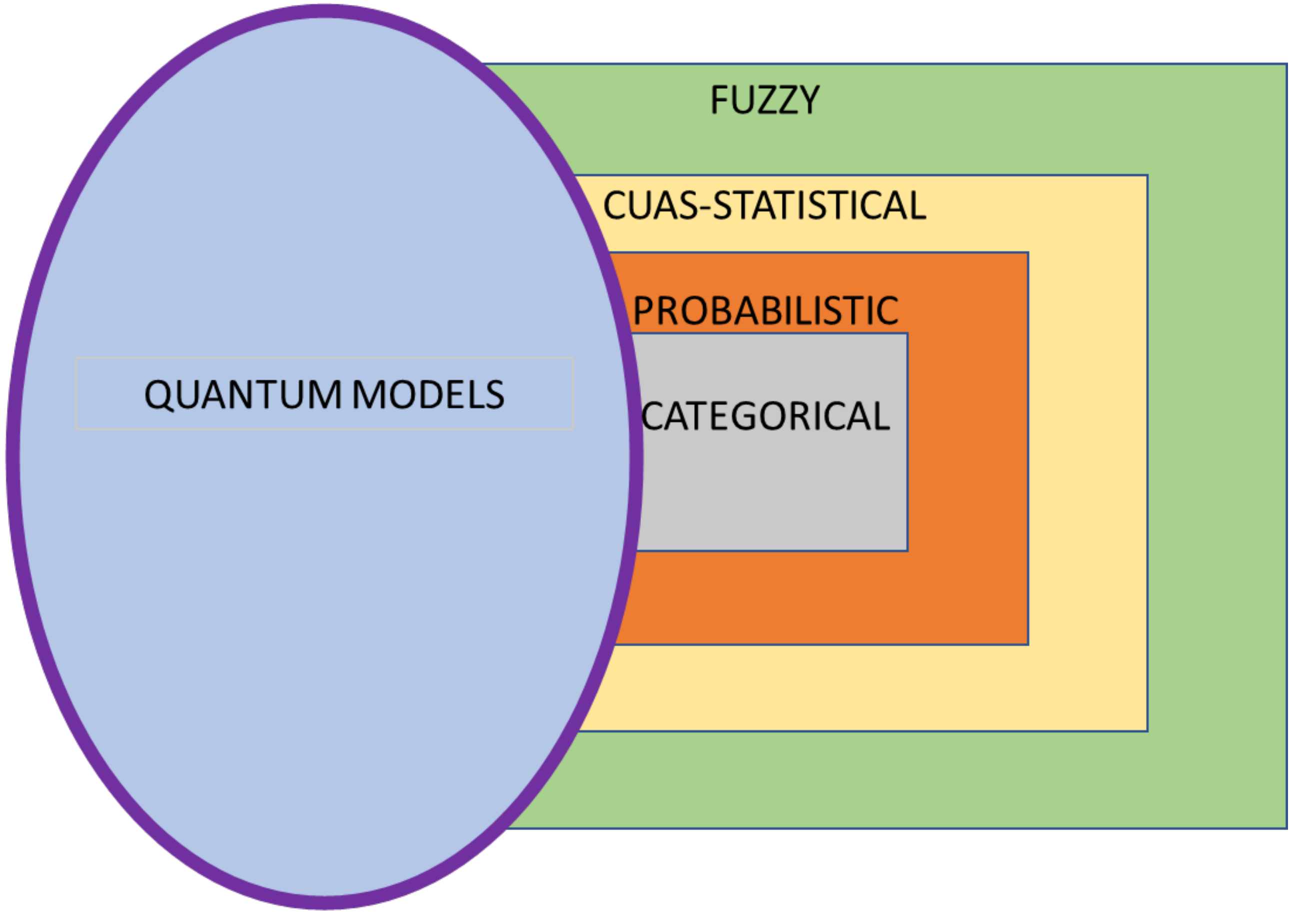
Normally the real world is non-deterministic. Intelligent systems are not always governed by deterministic laws, so that general laws are not always applicable.

Our model is often incomplete. There are many phenomena whose cause is unknown.

The lack of agreement between experts in the same field is frequent. Both circumstances make it difficult to include knowledge in a Rule-Based System.

REASONING WITH INACCURATE INFORMATION

- CATEGORICAL MODELS
- PROBABILISTIC MODELS
- CUASI-PROBABILISTIC MODELS
  - CERTAINTY FACTORS
  - THEORY OF EVIDENCE
- FUZZY LOGIC
- VECTORIAL APPROACHES
- QUANTUM MODELS



CLASSIC CATEGORICAL MODEL

DOMAIN:

- X = {ALL POSSIBLE FACTS} = {X(1), X(2),…, X(n)}
- Y = {ALL POSSIBLE INTERPRETATIONS} = {Y(1), Y(2),…, Y(m)}

PROBLEM:

- f (X)
  - f [X(i)] = 0 if X(i) is not a manifestation of my problem
  - f [X(i)] = 1 if X(i) is a manifestation of my problem
- g (Y)
  - g [Y(j)] = 0 if Y(j) is not a possible solution of my problem
  - g [Y(j)] = 1 if Y(j) is a possible solution of my problem

KNOWLEDGE

- E = E (X , Y)

LOGICAL PROBLEM

- E (f $\rightarrow$ g)

CLASSIC CATEGORICAL MODEL

- DOMAIN:
  - X = {X(1) , X(2)}
  - Y = {Y(1) , Y(2)}
- KNOWLEDGE
  - RULE 1: Y (2) $\rightarrow$ X (1)
  - RULE 2: Y (1) and not Y (2) $\rightarrow$ X (2)
  - RULE 3: not Y (1) and Y (2) $\rightarrow$ not X (2)
  - RULE 4: X (1) or X (2) $\rightarrow$ Y (1) or Y (2)
- PROBLEM
  - f (X) = not X (1) and X (2)
- SOLUTION
  - g (Y) = Y (1) and not Y (2)

ALTERNATIVE CATEGORICAL MODEL

	X1	X2	X3	x4
X (1)	0	0	1	1
X (2)	0	1	0	1

ALTERNATIVE CATEGORICAL MODEL

	Y1	Y2	Y3	Y4
Y (1)	0	0	1	1
Y (2)	0	1	0	1

ALTERNATIVE CATEGORICAL MODEL

	X1	X2	X3	X4	X1	X2	X3	X4	X1	X2	X3	X4	X1	X2	X3	X4
X (1)	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
X (2)	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
Y (1)	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
Y (2)	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
	Y1				Y2				Y3				Y4			

ALTERNATIVE CATEGORICAL MODEL

RULE 1: IF Y(2) THEN X(1)																
	X1	X2	X3	X4	X1	X2	X3	X4	X1	X2	X3	X4	X1	X2	X3	X4
X (1)	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
X (2)	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
Y (1)	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
Y (2)	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
	Y1				Y2				Y3				Y4			

ALTERNATIVE CATEGORICAL MODEL

RULE 2: IF Y(1) and not Y(2) THEN X(2)

	X1	X2	X3	X4	X1	X2	X3	X4	X1	X2	X3	X4	X1	X2	X3	X4
X (1)	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
X (2)	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
Y (1)	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
Y (2)	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
	Y1				Y2				Y3				Y4			

ALTERNATIVE CATEGORICAL MODEL

RULE 3: IF not Y(1) and Y(2) THEN not X(2)

	X1	X2	X3	X4	X1	X2	X3	X4	X1	X2	X3	X4	X1	X2	X3	X4
X (1)	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
X (2)	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
Y (1)	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
Y (2)	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
	Y1				Y2				Y3				Y4			

ALTERNATIVE CATEGORICAL MODEL

RULE 4: IF X(1) or X(2) THEN Y(1) or Y(2)																
	X1	X2	X3	X4	X1	X2	X3	X4	X1	X2	X3	X4	X1	X2	X3	X4
X (1)	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
X (2)	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
Y (1)	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
Y (2)	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
	Y1				Y2				Y3				Y4			

ALTERNATIVE CATEGORICAL MODEL

EXPANDED LOGICAL BASIS: X1Y1, X1Y2, X1Y3, X1Y4, X2Y1, X2Y2, X2Y3, X2Y4, X3Y1, X3Y2, X3Y3, X3Y4, X4Y1, X4Y2, X4Y3, X4Y4

REDUCED LOGICAL BASIS: X1Y1, X3Y2, X2Y3, X4Y3, X3Y4, X4Y4

OUR CURRENT PROBLEM: f (X) = not X(1) and X(2) $\rightarrow$ X2

ASSOCIATED COMPLEX IN RBL: X2Y3

SOLUTION: Y3 $\rightarrow$ g (Y) = Y(1) and not Y(2)

ALTERNATIVE CATEGORICAL MODEL

- UNCERTAINTY APPEARS SPONTANEOUSLY EVEN WHEN WE USE CATEGORICAL MODELS
  - Assume that f (X) = X(1) and not X(2) = X3
  - Associated complex in RLB = $\begin{cases} X3Y2 \\ X3Y4 \end{cases}$
  - TWO SOLUTIONS:
    - g1 (Y) = Y2 $\rightarrow$ not Y(1) and Y(2)
    - g2 (Y) = Y4 $\rightarrow$ Y(1) and Y(2)
  - INTERPRETATION
    - Y(1) could be a solution or could not be a solution
    - Y(2) is, for sure, a solution

PROBABILISTIC APPROACH

- BAYESIAN METHOD
  - A simple equation for Bayes Theorem

$$P\left({}^{Y}/_{X}\right) = \frac{P\left({}^{X}/_{Y}\right) \times P(Y)}{P(X)}$$

$$P\left({}^{X}/_{Y}\right) = \frac{P\left({}^{Y}/_{X}\right) \times P(X)}{P(Y)}$$

  - Generalization

$$P\left({}^{Y_0}/_{X}\right) = \frac{P\left({}^{X}/_{Y_0}\right) \times P(Y_0)}{\sum_i P\left({}^{X}/_{Y_i}\right) \times P(Y_i)}$$

$$P\left({}^{X_0}/_{Y}\right) = \frac{P\left({}^{Y}/_{X_0}\right) \times P(X_0)}{\sum_i P\left({}^{Y}/_{X_i}\right) \times P(X_i)}$$

PROBABILISTIC APPROACH

- JOINT PROBABILITIES OF ELEMENTS THAT DO NOT APPEAR IN THE REDUCED LOGICAL BASIS ARE ZERO.
- TO ENSURE MATHEMATICAL CONSISTENCY THE SUM OF THE CONDITIONAL PROBABILITIES OF ELEMENTS THAT SHARE CONDITIONS MUST BE ONE.
- P(Y2/X1) MUST BE ZERO SINCE X1Y2 DO NOT BELONG TO BLR
- P(Y3/X2) MUST BE ONE SINCE THERE IS ONLY ONE COMPLEX IN BLR X2
- P(Y2/X3) + P(Y4/X3) MUST BE ONE SINCE BOTH X3Y2 and X3Y4 ARE IN BLR
- THIS OPENS A VERY INTERESTING PERSPECTIVE FROM THE POINT OF VIEW OF KNOWLEGE ENGINEERING.
- OBVIOUSLY, CATEGORICAL REASONING IS A PARTICULAR CASE OF PROBABILISTIC REASONING.

Some Problems

GENERAL CONCEPTS:

- IMPRECISION concerns declarative knwowledge or FACTS
- UNCERTAINTY concerns procedural knowledge or RULES
- There are other kinds of inaccuracy.
  - Lack of knowledge
  - Vagueness
  - Belief
  - Disbelief
  - . . .

Some Problems

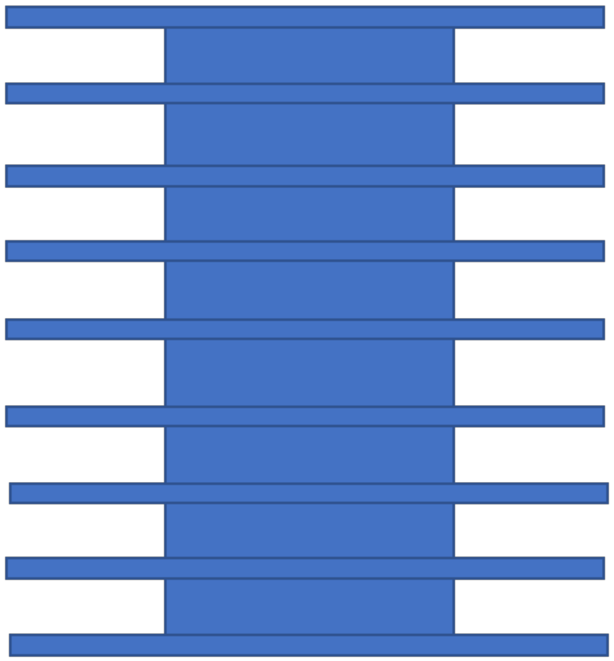
- When dealing with knowledge, one of the main problems of Probabilistic Models is their own mathematical consistency.
- $P(H/E) = X \; such \; that \;\; X \in [0,1]$
- $P(not \; H/E) = 1 - X \; such \; that \;\; X \in [0,1]$
- This is not always true, specially if our knowledge is not complete.
- Subjective Conditional Probilities have been proposed for this problem.

CERTAINTY FACTORS

- INCREASING BELIEF: $MB(H,E) = \frac{P(H/E)-P(H)}{1-P(H)}$
- INCREASING DISBELIEF: $MD(H,E) = \frac{P(H)-P(H/E)}{P(H)}$
- CERTAINTY FACTORS: $CF(H,E) = MB(H,E) - MD(H,E)$
- IF P(H/E) > P(H) THEN MB(H,E) > 0 and MD(H,E) = 0 → CF(H,E) = MB(H,E)
- IF P(H/E) < P(H) THEN MB(H,E) = 0 and MD(H,E) > 0 → CF(H,E) = - MD(H,E)
- IF P(H/E) = P(H) THEN MB(H,E) = 0 and MD(H,E) = 0 → CF(H,E) = 0

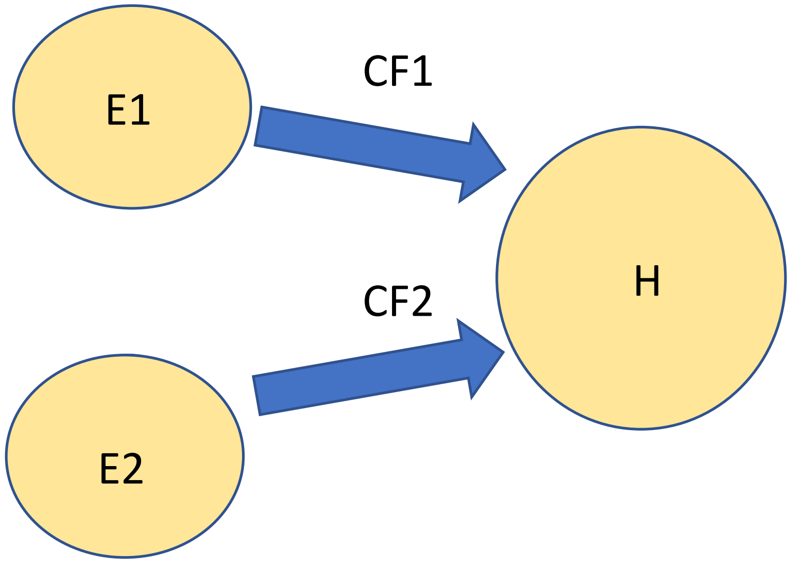
CERTAINTY FACTORS

SURE	CF = 1.00
ALMOST SURE	CF = 0.75
PROBABLY YES	CF = 0.50
PERHAPS	CF = 0.25
I DON'T KNOW	CF = 0.00
I DON'T THINK SO	CF = - 0.25
PROBABLY NOT	CF = - 0.50
ALMOST IMPOSSIBLE	CF = - 0.75
IMPOSSIBLE	CF = - 1.00



CERTAINTY FACTORS

- COMBINING EVIDENCES



1. CF1 > 0 and CF2 > 0
   - CF12 = (CF1 + CF2) – (CF1 x CF2)
2. CF1 < 0 and CF2 < 0
   - CF12 = (CF1 + CF2) + (CF1 x CF2)
3. CF1 x CF2 < 0
   - $CF12 = \frac{CF1+CF2}{1-\min\{|CF1|,|CF2|\}}$

CERTAINTY FACTORS

PROPAGATING INACCURACY

RULE_1: IF IT RAINS A LOT I NORMALLY STAY AT HOME:

$E1 \rightarrow H1$ with CF1

RULE_2: IF I STAY AT HOME I USE TO READ A BOOK:

$H1 \rightarrow H2$ with CF2

FACT_1: IT RAINS MODERATELY: E is not E1, but it is something similar:

$E \rightarrow E1$ with CF(It rains moderately)

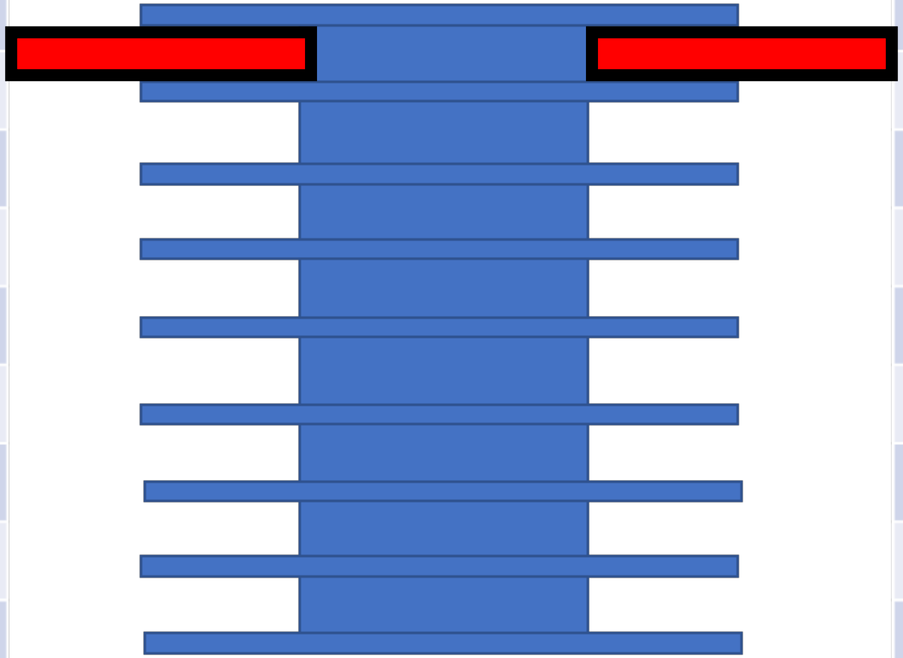
QUESTION: WHICH IS THE CERTAINTY FACTOR OF “READING A BOOK”?

CF(Reading a book, It rains moderately) =

CF(It rains moderately) x max {0, (CF1 x CF2)}

CERTAINTY FACTORS EXAMPLE

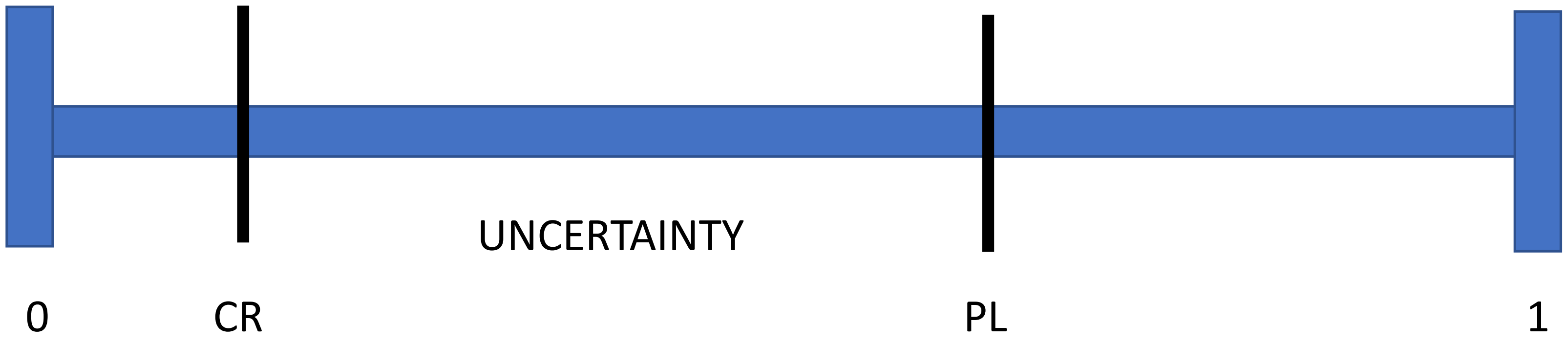
SURE
ALMOST SURE
PROBABLY YES
PERHAPS
I DON'T KNOW
I DON'T THINK SO
PROBABLY NOT
ALMOST IMPOSSIBLE
IMPOSSIBLE



CF = 1.00
CF = 0.75
CF = 0.50
CF = 0.25
CF = 0.00
CF = - 0.25
CF = - 0.50
CF = - 0.75
CF = - 1.00

THE THEORY OF EVIDENCE

- $CREDIBILITY = CR = \sum_{B \subset A} m(B)$
- $PLAUSIBILITY = PL = \sum_{B \cap A \neq \emptyset} m(B)$
- $CONFIDENCE\ INTERVAL = CI = [CR, PL]$



THE THEORY OF EVIDENCE

- $0 \leq Cr(A) \leq 1$
- $0 \leq Pl(A) \leq 1$
- $Cr(A) \leq Pl(A)$
- $Cr(A) \leq P(A) \leq Pl(A)$

$Cr(A) = 0$ y $Pl(A) = 1 \rightarrow$???

$Cr(A) = 1$ y $Pl(A) = 1 \rightarrow$ YES

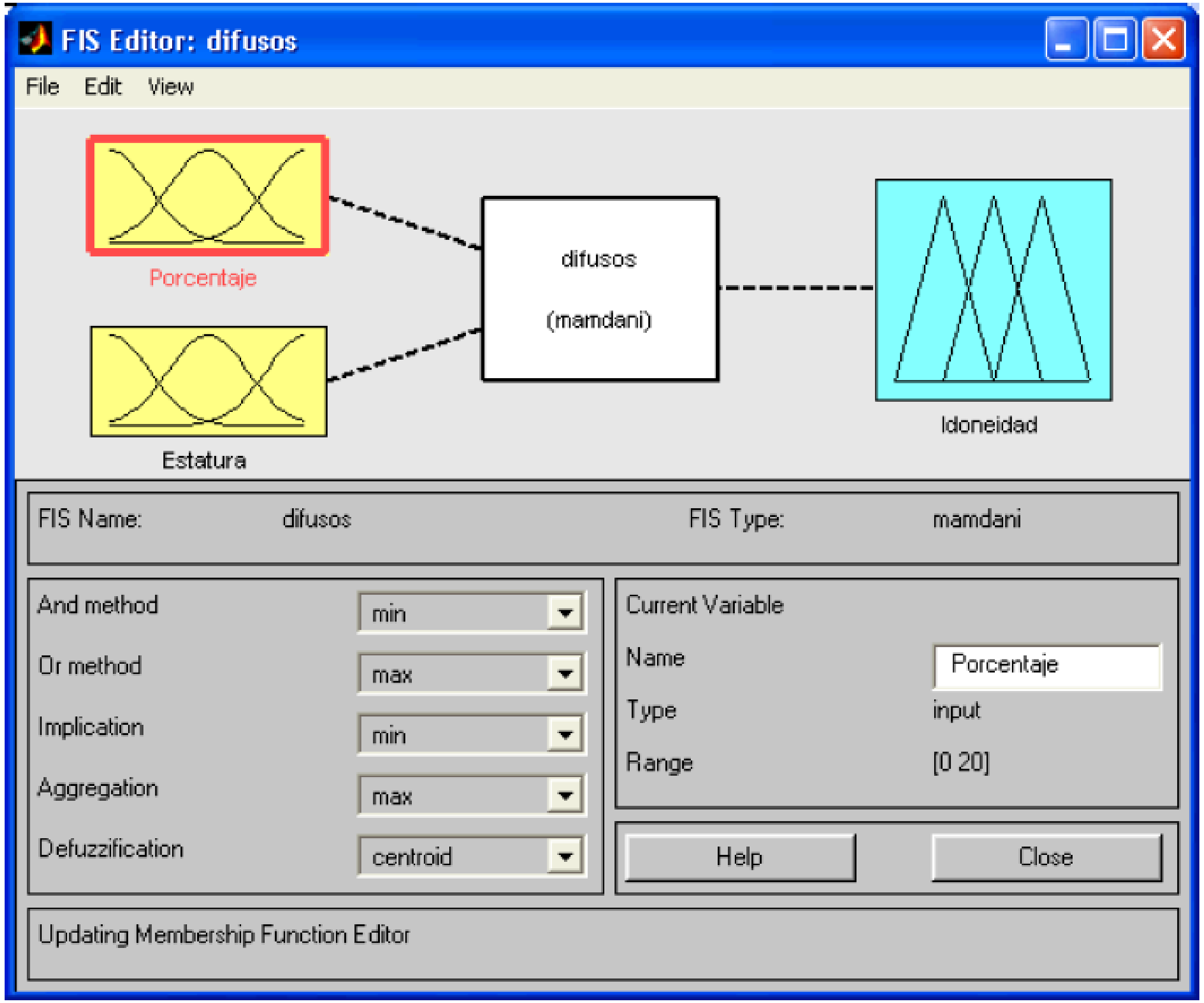
$Cr(A) = 0$ y $Pl(A) = 0 \rightarrow$ NO

CERTAINTY FACTORS ARE IN EVIDENTIAL THEORY

- CF(H,E) = s1 + s2 –s1s2

- e1 : A / m1(A) = s1 , m1(θ) = 1-s1
- e2 : A / m2(A) = s2 , m2(θ) = 1-s2
- m12(A) = s1s2 + s1(1-s2) + s2(1-s1) = s1 + s2 –s1s2

FUZZY LOGIC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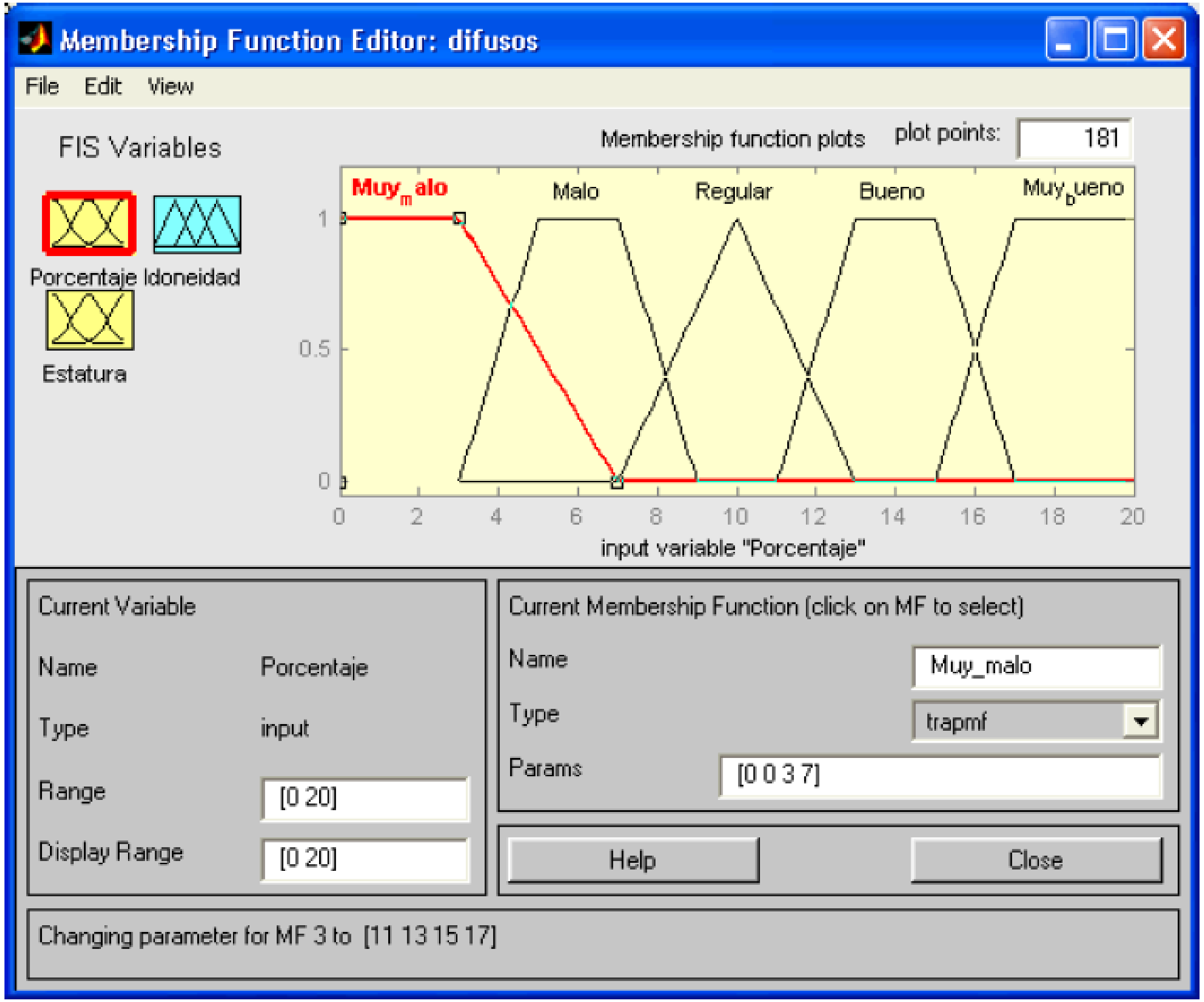
Updating Membership Function Editor

FUZZY LOGIC



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Current Variable
Name
Porcentaje
Type
input
Range
[0 20]
Display Range
[0 20]
Current Membership Function (click on MF to select)
Name
Muy_malo
Type
trapmf
Params
[0 0 3 7]
Help
Close
Changing parameter for MF 3 to [11 13 15 17]

FUZZY LOGIC

- General features
- Nomenclature
- Algebraic structure
- Algebraic operations
- Knowledge representation
- Fuzzy reasoning

FUZZY LOGIC

- Linguistic criteria
- Ambiguity
- Language
- Classification
- Taxonomies
- Hierarchies
- Reasoning

FUZZY LOGIC

- Concepts
- Definitions
- Plant? Stone? Virus?
- Subjective issues
- Contexts

FUZZY LOGIC

Lofti Zadeh, 1965 Universe N of natural numbers

$A \subseteq N$

A = Natural numbers, even numbers and less than 10

A = {2,4,6,8} such that 2 $\in A$ *and* 3 $\notin A$

FUZZY LOGIC

- U = Living human beings
- A $\subseteq$ U
- A = Tall dark-haired living human beings
- Conventional sets:
- $U, A \subseteq U \text{ such that } \exists\, \mu_A(x) : U \to \{0, x \notin A : 1, x \in A\}\ \forall x \in U$
- Fuzzy sets:
- $U, A \subseteq U \text{ such that } \exists\, \mu_A(x) : U \to [0,1]\ \forall x \in U$

FUZZY LOGIC

Criteria for defining the membership degree function

U = Living people

A = Young living persons

FUZZY LOGIC

$\mu_A(x) = 1 : \forall x \,/\, AGE\,(x) \leq 25$

$\mu_A(x) = 0 : \forall x \,/\, AGE\,(x) \geq 65$

¿$\mu_A(x) : \forall x \,/\, 25 < AGE\,(x) < 65$?

FUZZY LOGIC

$$\mu A(x) = \frac{65 - AGE\ (x)}{40} : \forall x / AGE(x) \in [25,65]$$

A = *Young living person*

$AGE\ (Juan)$	$= 17$	$\rightarrow$	$\mu\ Young\ (Juan)$	$= 1.00$
$AGE\ (Marisa)$	$= 31$	$\rightarrow$	$\mu\ Young\ (Marisa)$	$= 0.85$
$AGE\ (Blas)$	$= 47$	$\rightarrow$	$\mu\ Young\ (Blas)$	$= 0.45$
$AGE\ (Ana)$	$= 57$	$\rightarrow$	$\mu\ Young\ (Ana)$	$= 0.20$
$AGE(Alex)$	$= 73$	$\rightarrow$	$\mu\ Young\ (Alex)$	$= 0.00$

FUZZY LOGIC

Juan	is	1.00	young
Marisa	is	0.85	young
Blas	is	0.45	young
Ana	is	0.20	young
Alex	is	0.00	young

FUZZY LOGIC

- 0.00 - 0.00 = *Is not*
- 0.00 - 0.20 = *Is very few*
- 0.20 - 0.40 = *Is a little*
- 0.40 - 0.60 = *Is somewhat*
- 0.60 - 0.80 = *Is moderately*
- 0.80 - 1.00 = *Is quite*
- 1.00 - 1.00 = *Is*

FUZZY LOGIC

$x \in U$	LABEL (μ)	$A \subseteq U$
Juan	IS	YOUNG
Marisa	IS QUITE	YOUNG
Blas	IS SOMEWHAT	YOUNG
Ana	IS VERY FEW	YOUNG
Alex	IS NOT	YOUNG

FUZZY LOGIC

- Comments
  - The diffuse zone does not have to be linear
  - The linguistic scale is arbitrary
  - The number of items in the scale is arbitrary
  - We can define complementary fuzzy sets
  - Any set can be fuzzyfied

FUZZY LOGIC

$$\forall A \subset U : A\, is\, Fuzzy \leftrightarrow \exists \mu_A(x) : U \rightarrow [0,1] \forall x \in U$$

FUZZY LOGIC

$$Universe : U$$

$$Z \subset U / \exists \mu_Z(x) : U \rightarrow [0,1] \forall x \in U$$

$$Z = \varnothing \leftrightarrow \mu_Z(x) = 0 \forall x \in U$$

FUZZY LOGIC

$$Universe: U$$

$$A \subset U / \exists \mu_A(x): U \rightarrow [0,1] \forall x \in U$$

$$B \subset U / \exists \mu_B(x): U \rightarrow [0,1] \forall x \in U$$

$$A = B \leftrightarrow \mu_A(x) = \mu_B(x): \forall x \in U$$

FUZZY LOGIC

$$Universo\_referencial : U$$

$$A \subset U \,/\, \exists \mu_A(x) : U \rightarrow [0,1] \; \forall x \in U$$

$$\neg A = A' \leftrightarrow \mu_{A'}(x) = 1 - \mu_A(x) : \forall x \in U$$

FUZZY LOGIC

$$Universo\_referencial : U$$

$$A \subset U / \exists \mu_A(x) : U \rightarrow [0,1] \forall x \in U$$

$$B \subset U / \exists \mu_B(x) : U \rightarrow [0,1] \forall x \in U$$

$$B \subset A \leftrightarrow \mu_B(x) \leq \mu_A(x) : \forall x \in U$$

FUZZY LOGIC

$$U = \{1,2,3,4,5,6\}$$

$$A \subset U \,/\, A = \{1,2,3,4\}$$

$$B \subset U \,/\, B = \{1,3\}$$

$$\mu_A(x) = \{1/1 + 1/2 + 1/3 + 1/4 + 0/5 + 0/6\}$$

$$\mu_B(x) = \{1/1 + 0/2 + 1/3 + 0/4 + 0/5 + 0/6\}$$

$$1/1 = 1/1 : 1/2 > 0/2 : 1/3 = 1/3$$

$$1/4 > 0/4 : 0/5 = 0/5 : 0/6 = 0/6$$

FUZZY LOGIC

$$Universo\_referencial : U$$

$$A \subset U / \exists \mu_A(x) : U \rightarrow [0,1] \forall x \in U$$

$$B \subset U / \exists \mu_B(x) : U \rightarrow [0,1] \forall x \in U$$

$$C \subset U / \exists \mu_C(x) : U \rightarrow [0,1] \forall x \in U$$

$$C = A \cup B \leftrightarrow \mu_C(x) = \max\{\mu_A(x), \mu_B(x)\} : \forall x \in U$$

$$Asociatividad : A \cup (B \cup C) = (A \cup B) \cup C$$

FUZZY LOGIC

$$Universo\_referencial : U$$

$$A \subset U / \exists \mu_A(x) : U \rightarrow [0,1] \forall x \in U$$

$$B \subset U / \exists \mu_B(x) : U \rightarrow [0,1] \forall x \in U$$

$$C \subset U / \exists \mu_C(x) : U \rightarrow [0,1] \forall x \in U$$

$$C = A \cap B \leftrightarrow \mu_C(x) = \min\{\mu_A(x), \mu_B(x)\} : \forall x \in U$$

$$Asociatividad : A \cap (B \cap C) = (A \cap B) \cap C$$

FUZZY LOGIC

$Universo\ \_referencia\ \ 1 : U$

$A \subset U \,/\, \exists \mu_A(x) : U \rightarrow [0,1] \forall x \in U$

$B \subset U \,/\, \exists \mu_B(x) : U \rightarrow [0,1] \forall x \in U$

1^a *Ley*

$\neg(A \cup B) = \neg A \cap \neg B$

$\neg[\mu_A(x) \vee \mu_B(x)] = [\neg \mu_A(x) \wedge \neg \mu_B(x)] : \forall x \in U$

2^a *Ley*

$\neg(A \cap B) = \neg A \cup \neg B$

$\neg[\mu_A(x) \wedge \mu_B(x)] = [\neg \mu_A(x) \vee \neg \mu_B(x)] : \forall x \in U$

FUZZY LOGIC

$$Universo\_referencial : U$$

$$A \subset U / \exists \mu_A(x) : U \rightarrow [0,1] \forall x \in U$$

$$B \subset U / \exists \mu_B(x) : U \rightarrow [0,1] \forall x \in U$$

$$\neg(A \cup B) \rightarrow \mu_{\neg(A \cup B)}(x) = 1 - \max\{\mu_A(x), \mu_B(x)\} : \forall x \in U$$

$$\neg A \cap \neg B \rightarrow \mu_{\neg A \cap \neg B}(x) = \min\{1 - \mu_A(x), 1 - \mu_B(x)\} : \forall x \in U$$

FUZZY LOGIC

$Universo\_referencial\_U$

$A \subset U / \exists \mu_A(x) \rightarrow U:[0,1] \forall x \in U$

$B \subset U / \exists \mu_B(x) \rightarrow U:[0,1] \forall x \in U$

$C \subset U / \exists \mu_C(x) \rightarrow U:[0,1] \forall x \in U$

1^a_ley

$C \cap (A \cup B) = (C \cap A) \cup (C \cap B)$

$\mu_C(x) \wedge [\mu_A(x) \vee \mu_B(x)] = [\mu_C(x) \wedge \mu_A(x)] \vee [\mu_C(x) \wedge \mu_B(x)]: \forall x \in U$

2^a_ley

$C \cup (A \cap B) = (C \cup A) \cap (C \cup B)$

$\mu_C(x) \vee [\mu_A(x) \wedge \mu_B(x)] = [\mu_C(x) \vee \mu_A(x)] \wedge [\mu_C(x) \vee \mu_B(x)]: \forall x \in U$

FUZZY LOGIC

$$Casos\_extremos$$

$$\mu_A(x) \geq \mu_B(x) : \forall x \in U$$

$$1 - \max\{\mu_A(x), \mu_B(x)\} = 1 - \mu_A(x) : \forall x \in U$$

$$\min\{1 - \mu_A(x), 1 - \mu_B(x)\} = 1 - \mu_A(x) : \forall x \in U$$

$$\mu_A(x) \leq \mu_B(x) : \forall x \in U$$

$$1 - \max\{\mu_A(x), \mu_B(x)\} = 1 - \mu_B(x) : \forall x \in U$$

$$\min\{1 - \mu_A(x), 1 - \mu_B(x)\} = 1 - \mu_B(x) : \forall x \in U$$

FUZZY LOGIC

$Ley\_del\_tercero\_excluido$

$Universo\_referencial\_U$

$Conjuntos\_ordinarios : A \cup \neg A = U$

$Conjuntos\_difusos :$

$$A \subset U / \exists \mu_A(x) \rightarrow U : [0,1] \forall x \in U$$

$$\neg A \rightarrow \mu_{A'}(x) = 1 - \mu_A(x) : \forall x \in U$$

$$A \cup \neg A \rightarrow \mu_{A \cup A'}(x) = \max\{\mu_A(x), \mu_{A'}(x)\} = \max\{\mu_A(x), 1 - \mu_A(x)\} : \forall x \in U$$

$$\mu_{A \cup A'}(x) \geq \frac{1}{2}$$

FUZZY LOGIC

$$Ley\_de\_no\_contradicción$$

$$Universo\_referencial\_U$$

$$Conjuntos\_ordinarios: A \cap \neg A = \varnothing$$

$$Conjuntos\_difusos:$$

$$A \subset U / \exists \mu_A(x) \rightarrow U:[0,1] \forall x \in U$$

$$\neg A \rightarrow \mu_{A'}(x) = 1 - \mu_A(x) : \forall x \in U$$

$$A \cap \neg A \rightarrow \mu_{A \cap A'}(x) = \min\{\mu_A(x), \mu_{A'}(x)\} = \min\{\mu_A(x), 1 - \mu_A(x)\} : \forall x \in U$$

$$\mu_{A \cap A'}(x) \leq \frac{1}{2}$$

FUZZY LOGIC

$$Universo\_referencial\_U$$

$$A \subset U / \exists \mu_A(x) \rightarrow U :[0,1] \forall x \in U$$

$$B \subset U / \exists \mu_B(x) \rightarrow U :[0,1] \forall x \in U$$

$$A \times B \rightarrow \mu_{AB}(x) = \mu_A(x) \times \mu_B(x) : \forall x \in U$$

FUZZY LOGIC

$$Conjuntos\_ordinarios$$

$$A \times B = A \cap B$$

$$Conjuntos\_difusos$$

$$Universo\_referencia\ l\_U$$

$$A \subset U \,/\, \exists \mu_A(x) \rightarrow U : [0,1] \forall x \in U$$

$$B \subset U \,/\, \exists \mu_B(x) \rightarrow U : [0,1] \forall x \in U$$

$$A \times B \rightarrow \mu_{AB}(x) = \mu_A(x) \times \mu_B(x) : \forall x \in U$$

$$A \cap B \rightarrow \mu_{A \cap B}(x) = \min\{ \mu_A(x), \mu_B(x)\} : \forall x \in U$$

$$\mu_{AB}(x) < \min\{ \mu_A(x), \mu_B(x)\} : \forall x \in U$$

$$A \times B \subset A \cap B$$

FUZZY LOGIC

$$Universo\_referencial\_U$$

$$A \subset U / \exists \mu_A(x) \rightarrow U :[0,1] \forall x \in U$$

$$B \subset U / \exists \mu_B(x) \rightarrow U :[0,1] \forall x \in U$$

$$A + B \rightarrow \mu_{A+B}(x) = \mu_A(x) + \mu_B(x) : \forall x \in U$$

$$A \oplus B \rightarrow \mu_{A \oplus B}(x) = \min\{1, \mu_A(x) + \mu_B(x)\} : \forall x \in U$$

FUZZY LOGIC

$$Universo\_referencial\_U$$

$$A \subset U / \exists \mu_A(x) \to U :[0,1] \forall x \in U$$

$$B \subset U / \exists \mu_B(x) \to U :[0,1] \forall x \in U$$

$$A - B \to \mu_{A-B}(x) = \mu_A(x) - \mu_B(x) : \forall x \in U$$

$$|A - B| \to \mu_{|A-B|}(x) = |\mu_A(x) - \mu_B(x)| : \forall x \in U$$

FUZZY LOGIC

$$Universo\_referencial\_U$$

$$A \subset U / \exists \mu_A(x) \rightarrow U:[0,1] \forall x \in U$$

$$Núcleo: N_A = \{x \in U / \mu_A(x) = 1\}$$

$$A = Normalizado \leftrightarrow N_A \neq \{\emptyset\}$$

FUZZY LOGIC

Given a referential U, we define a fuzzy relation of order “n” on U, as a fuzzy set A in the space U x U x…x U (n times), characterized by a membership degree function of the type:

$$\mu_A(x1,...,xn) : \forall x \in U$$

FUZZY LOGIC

	U2			
U1	1	2	3	4
1	1.0	0.8	0.2	0.0
2	0.8	1.0	0.8	0.2
3	0.2	0.8	1.0	0.8
4	0.0	0.2	0.8	0.0

- Referential U = {1, 2, 3, 4}
- Fuzzy relation of order 2
- A = {approximately equal numbers}

FUZZY LOGIC

Rule: 1

IF x is A3
OR y is B1
THEN z is C1

Rule: 2

IF x is A2
AND y is B2
THEN z is C2

Rule: 3

IF x is A1
THEN z is C3

Rule: 1

IF project_funding is adequate
OR project_staffing is small
THEN risk is low

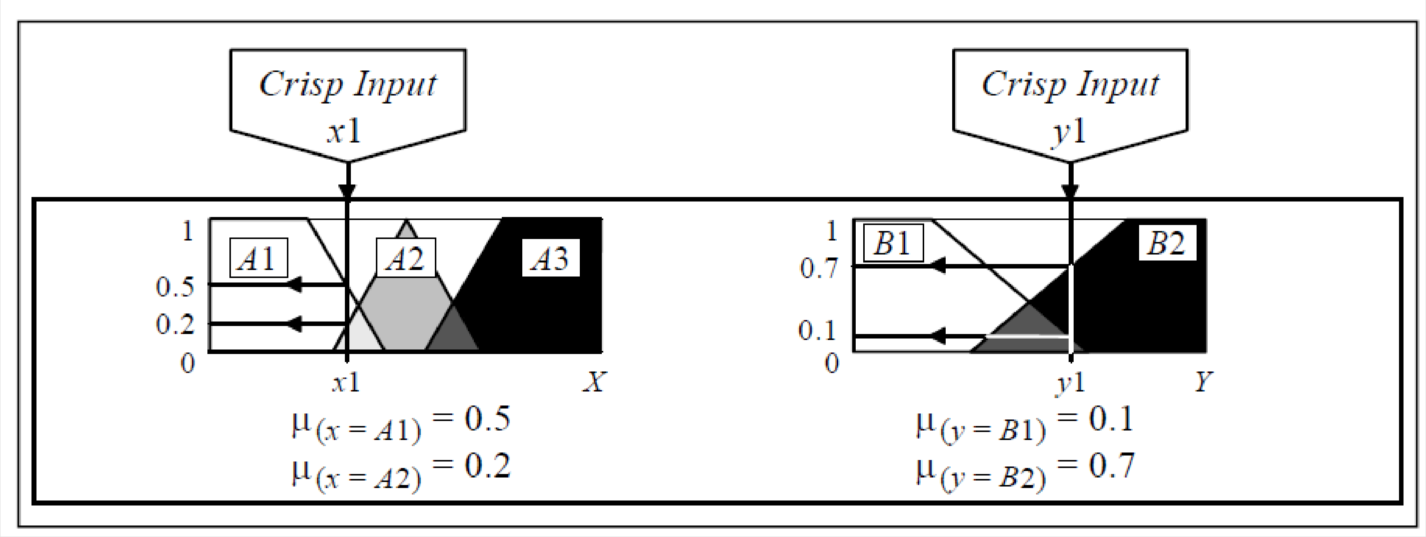
Rule: 2

IF project_funding is marginal
AND project_staffing is large
THEN risk is normal

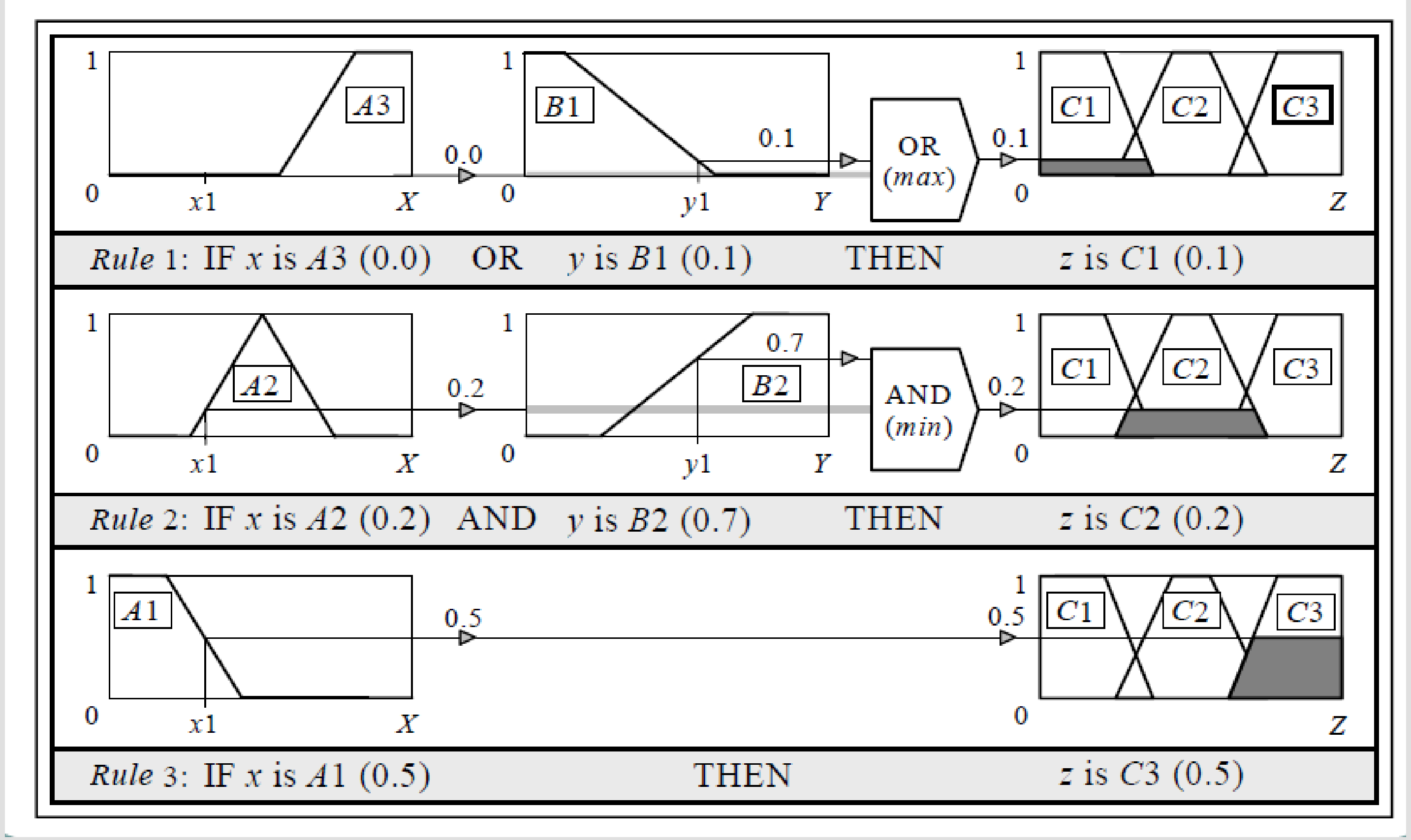
Rule: 3

IF project_funding is inadequate
THEN risk is high

FUZZY LOGIC

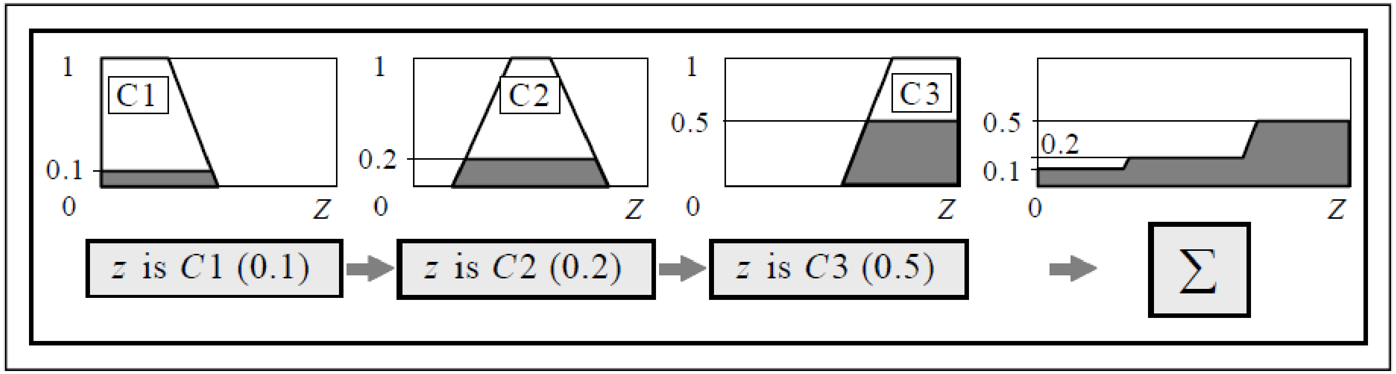

Crisp Input
x1
Crisp Input
y1
1
0.5
0.2
0
A1
A2
A3
x1
X
μ(x = A1) = 0.5
μ(x = A2) = 0.2
1
0.7
0.1
0
B1
B2
y1
Y
μ(y = B1) = 0.1
μ(y = B2) = 0.7

FUZZY LOGIC



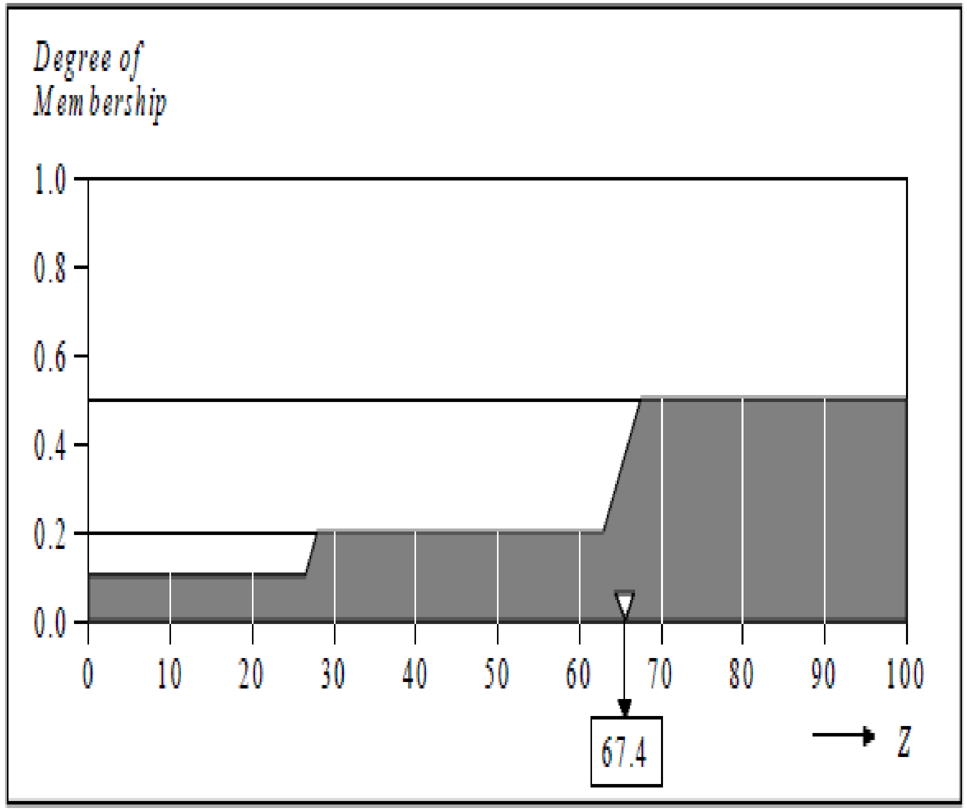
A3
x1
X
0.0
B1
0.1
y1
Y
OR (max)
0.1
C1
C2
C3
Z
Rule 1: IF x is A3 (0.0) OR y is B1 (0.1) THEN z is C1 (0.1)
A2
0.2
0.7
B2
AND (min)
Rule 2: IF x is A2 (0.2) AND y is B2 (0.7) THEN z is C2 (0.2)
A1
0.5
Rule 3: IF x is A1 (0.5) THEN z is C3 (0.5)

FUZZY LOGIC


1
C1
0.1
0
Z
1
C2
0.2
0
Z
1
C3
0.5
0
Z
0.5
0.2
0.1
0
Z
z is C1 (0.1)
z is C2 (0.2)
z is C3 (0.5)
Σ

FUZZY LOGIC

$$COG = \frac{\sum_{x=a}^{b} \mu_A(x)\,x}{\sum_{x=a}^{b} \mu_A(x)}$$



THANK YOU VERY MUCH FOR YOUR ATTENTION