Temporal Equilibrium Logic with Past Operators

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Essays dedicated to Luis Fariñas del Cerro

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Initial motivation

Knowledge
Representation

- During my PhD (1990's), I was interested in KR for dynamic domains: reasoning about actions and change
- Representational problems: frame, Yale Shooting, ...
  How to deal with defaults like inertia?
- Most approaches used First Order Logic to represent time: Situation Calculus, Event Calculus, etc
- But I also became interested in temporal modal approaches.
  I downloaded many papers by some Fariñas del Cerro (French? Galician?)

Fariñas del Cerro?

French coauthors, Toulouse, . . .

But did you know about his Galician Connection?

Ferrol, Galicia, Spain

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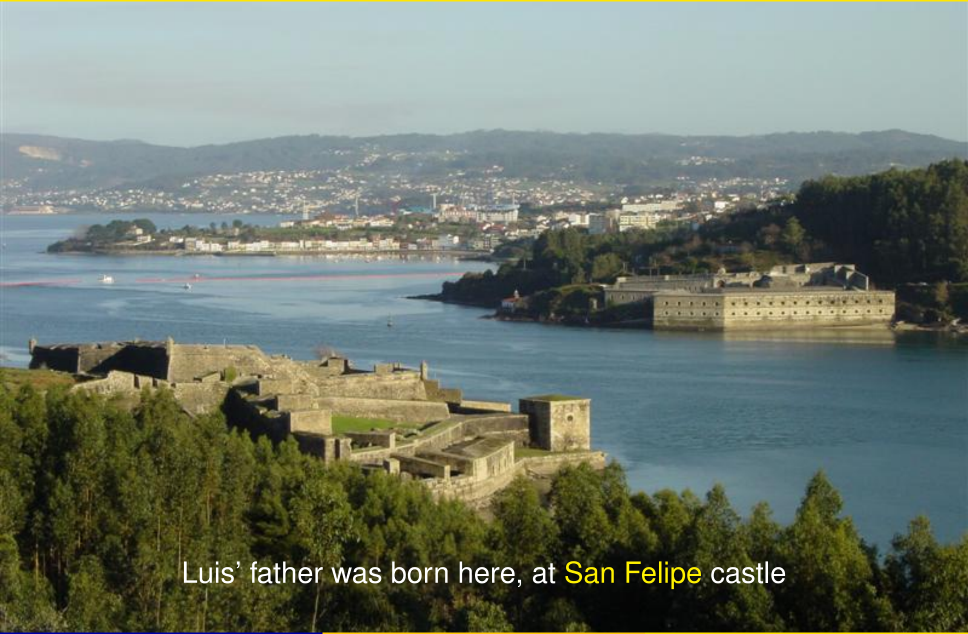


Luis' father was born here, at San Felipe castle

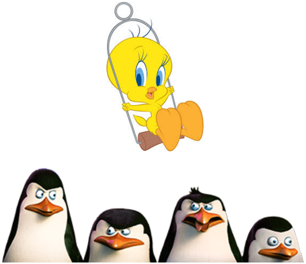
Initial motivation

- In 1996? I found out that Luis himself was teaching an introductory course on modal logic at Santiago de Compostela

- Fantastic experience: modal logic rocks!

Logic Programming and NMR

(Back to actions and change) The stress was put on Non-monotonic Reasoning (NMR)



- You said it's a penguin?
- Well, it is not flying ...

[AIJ 1980] Circumscription, Default Logic, NM Modal logic

[Gelfond & Lifschitz, JLP 93] *Representing Action and Change by Logic Programs*

Transition systems in Answer Set Programming (ASP)

Transition systems in ASP

ASP = problem solving paradigm. Similar to SAT (models=solutions)

Time: integer variables (iteratively) grounded before solving

Some nice features

- Elaboration tolerance: small changes in the problem ⇒ small changes in representation
- Simple solution to frame, ramification and qualification problems
- Easy to switch reasoning task:
  prediction (or simulation), explanation, planning, diagnosis
- Simple (linear) time structure: integer argument in predicates
- Incremental ASP exploits time index to reuse grounding/solving

Transition systems in ASP

But not thought for temporal reasoning

- ✗ Planning by iterative deepening with finite path length:
  we cannot prove non-existence of plan
- ✗ Reactive systems out of the scope:
  e.g. a network server must keep on running (potentially) forever
- ✗ (Forgotten) reasoning task: verification of temporal properties.
  E.g. "*At some point, fluent p will never change again*"
- ✗ Existing formal methods for transition systems: outside ASP

Idea: temporal (modal) LP

- Idea: why not using a modal extension of LP?
- Modal LP: solid background and literature
  - MOLOG [Fariñas 86]: modal operators in Prolog
  - Linear Temporal Logic (LTL) + LP: [Gabbay 87, Abadi & Manna 89, Orgun & Wadge 92] etc.
  - Example: TEMPLOG [Abadi 89]. Rules like

$$\Box(p \leftarrow \bigcirc q \wedge \Diamond r)$$

- Problems with temporal LP formalisms
  - Good for goal-oriented top-down reading.
    Bad for representing causal rules
  - We cannot represent defaults: no default negation

Temporal Equilibrium Logic

- Equilibrium Logic [Pearce 96] captures stable models and ASP using an intermediate logic (*Here-and-There*)
- Idea: mixing temporal modalities with an intermediate logic. Example: intuitionistic modal logic [Fariñas & Raggio 83]
- Temporal Equilibrium Logic (TEL) = LTL + Equilibrium Logic [Cabalar& Vega 07]
- A pair of tools [Cabalar & Diéguez 11, 14] using model checking and automata transformations
- TEL defines temporal stable models for arbitrary LTL formulas.

$$\Box(\bigcirc \mathit{light} \leftarrow \mathit{push} \wedge \neg \mathit{broken})$$

Keypoint

- For practical KR most implications go from present to future. We called this syntactic fragment splittable theories:

$$\Box(\underbrace{\bigcirc \Diamond\, light}_{\text{head=future}} \leftarrow \underbrace{push \wedge \neg broken}_{\text{body=present}})$$

- Operators in the head may allow going beyond pure transitions
- But adding other operators in the rule body seems awkward:

$$\Box(\underbrace{parents\_not\_dating}_{1955} \leftarrow \underbrace{\Diamond travels\_back}_{1985})$$

Keypoint

- A more practical choice: rule conditions that inspect the past.
  Example: the first time it's pushed, we have a 2 tics delay

  $$\Box(\bigcirc\bigcirc \mathit{light} \quad \leftarrow \quad \mathit{push} \wedge \boxminus\neg \mathit{push})$$

  $\boxminus\neg \mathit{push}$ = not pushed before

- Of course, we can use auxiliary atoms to memorise past events

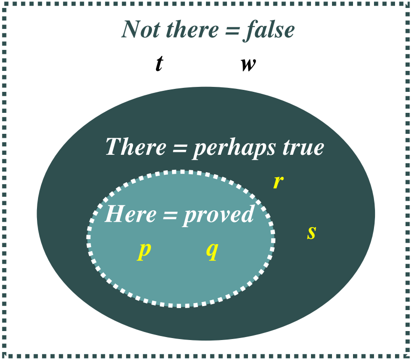
  $$\begin{array}{rcl} \Box(\bigcirc\bigcirc \mathit{light} & \leftarrow & \mathit{push} \wedge \neg \mathit{pushed\_before}) \\ \Box(\bigcirc \mathit{pushed\_before} & \leftarrow & \mathit{push}) \\ \Box(\bigcirc \mathit{pushed\_efore} & \leftarrow & \mathit{pushed\_before}) \end{array}$$

  But temporal past is exponentially more succint
  [Laroussinie et al 02].

- Paper goal: extend TEL with past operators

Equilibrium Logic

- Equilibrium Logic [Pearce96]: generalises stable models for arbitrary propositional theories.
- Here-and-There + selected models (classical & minimal)



When $H = T$ we have a classical model.

Here-and-There

Satisfaction of formulas

$\langle H, T\rangle \models \varphi \quad \Leftrightarrow \quad$ "φ is proved"
$\langle T, T\rangle \models \varphi \quad \Leftrightarrow \quad$ "φ potentially true" $\quad \Leftrightarrow \quad T \models \varphi$ classically

- $\langle H, T\rangle \models p$ if $p \in H$ (for any atom p)
- $\wedge, \vee$ as always
- $\langle H, T\rangle \models \varphi \rightarrow \psi$ if both
  - $T \models \varphi \rightarrow \psi$ classically
  - $\langle H, T\rangle \models \varphi$ implies $\langle H, T\rangle \models \psi$
- Negation $\neg F$ is defined as $F \rightarrow \perp$

Definition (Equilibrium/stable model)

A model $\langle T, T\rangle$ of Γ is an equilibrium model iff

there is no $H \subset T$ such that $\langle H, T\rangle \models \Gamma$.

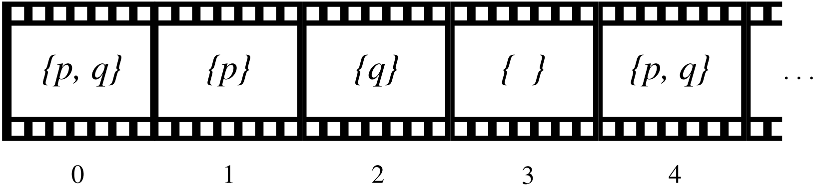
(Linear) Temporal Equilibrium Logic

- Syntax = propositional plus
  - $\Box\varphi$ = "forever" φ
  - $\Diamond\varphi$ = "eventually" φ
  - $\bigcirc\varphi$ = "next moment" φ
  - $\varphi\ \mathcal{U}\ \psi$ = φ "until eventually" ψ
  - $\varphi\ \mathcal{R}\ \psi$ = φ "release" ψ

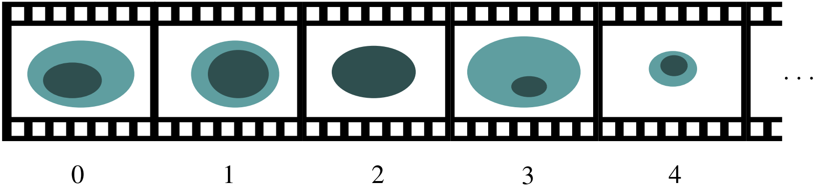
  In the paper: new operators
  - $\boxminus\varphi$ = "always held" φ
  - $\diamondminus\varphi$ = "in the past" φ
  - $\ominus\varphi$ = "previously" φ
  - $\varphi\ \mathcal{S}\ \psi$ = φ "since" ψ
  - $\varphi\ \mathcal{T}\ \psi$ = φ "triggered" ψ
- As we had with Equilibrium Logic:
  1. A monotonic underlying logic: Temporal Here-and-There (THT)
  2. An ordering among models. Select minimal models.

Sequences

- In standard LTL, interpretations are ∞ sequences of sets of atoms



- In THT we will have ∞ sequences of HT interpretations



- We define an ordering among sequences $\mathbf{H} \leq <\mathbf{T}$ when

$$
\begin{array}{ccccccccccc}
T_0 & \longrightarrow & T_1 & \longrightarrow & T_2 & \longrightarrow & \ldots & \longrightarrow & T_i & \longrightarrow & \ldots \\
\cup| & & \cup| & & \cup|\cup & & & & \cup| & & \\
H_0 & \longrightarrow & H_1 & \longrightarrow & H_2 & \longrightarrow & \ldots & \longrightarrow & H_i & \longrightarrow & \ldots
\end{array}
$$

Definition (THT-interpretation)

is a pair of sequences of sets of atoms $\langle \mathbf{H}, \mathbf{T} \rangle$ with $\mathbf{H} \leq \mathbf{T}$. □

Temporal Here-and-There (THT)

$\langle \mathbf{H}, \mathbf{T} \rangle, i \models \varphi \quad \Leftrightarrow \quad$ "φ is proved at i"
$\langle \mathbf{T}, \mathbf{T} \rangle, i \models \varphi \quad \Leftrightarrow \quad$ "φ potentially true at i" $\quad \Leftrightarrow \quad \mathbf{T}, i \models \varphi$ in LTL

- An interpretation $M = \langle \mathbf{H}, \mathbf{T} \rangle$ satisfies α at situation i, written $M, i \models \alpha$

α	$M, i \models \alpha$ when ...
an atom p	$p \in H_i$
$\wedge, \vee$	as usual
$\varphi \rightarrow \psi$	$\mathbf{T}, i \models \varphi \rightarrow \psi$ in LTL and $\langle \mathbf{H}, \mathbf{T} \rangle, i \models \varphi$ implies $\langle \mathbf{H}, \mathbf{T} \rangle, i \models \psi$

$\langle \mathbf{H}, \mathbf{T} \rangle, i \models \varphi \quad \Leftrightarrow \quad$ "φ is proved at i"

$\langle \mathbf{T}, \mathbf{T} \rangle, i \models \varphi \quad \Leftrightarrow \quad$ "φ potentially true at i" $\quad \Leftrightarrow \quad \mathbf{T}, i \models \varphi$ in LTL

- An interpretation $M = \langle \mathbf{H}, \mathbf{T} \rangle$ satisfies α at situation i, written $M, i \models \alpha$

α	$M, i \models \alpha$ when ...
$\bigcirc\varphi$	$(M, i+1) \models \varphi$
$\ominus\varphi$	$i = 0$ or $i > 0, (M, i-1) \models \varphi$
$\hat{\ominus}\varphi$	$i > 0$ and $(M, i-1) \models \varphi$
$\square\varphi$	$\forall j \geq i$ such that $M, j \models \varphi$
$\boxminus\varphi$	$\forall j, 0 \leq j \leq i$ such that $M, j \models \varphi$
$\Diamond\varphi$	$\exists j \geq i$ such that $M, j \models \varphi$
$\diamondminus\varphi$	$\exists j, 0 \leq j \leq i$ such that $M, j \models \varphi$
$\vdots$	$\vdots$

Temporal Equilibrium Models

- M is a model of a theory Γ when $M, 0 \models \alpha$ for all $\alpha \in \Gamma$
- Again, we fix potential truth and minimise proved conclusions

Definition (Temporal Equilibrium Model)

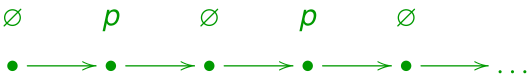
of a theory Γ is a model $M = \langle \mathbf{T}, \mathbf{T} \rangle$ of Γ such that there is no $\mathbf{H} < \mathbf{T}$ satisfying $\langle \mathbf{H}, \mathbf{T} \rangle, 0 \models \Gamma$. □

Some examples

- Example 1: TEL models of $\Box(\neg p \rightarrow \bigcirc p)$. It's like an infinite program:

$$
\begin{array}{rcl}
\neg p & \rightarrow & \bigcirc p \\
\neg \bigcirc p & \rightarrow & \bigcirc^2 p \\
\neg \bigcirc^2 p & \rightarrow & \bigcirc^3 p \\
& \vdots &
\end{array}
$$

- TEL models have the form



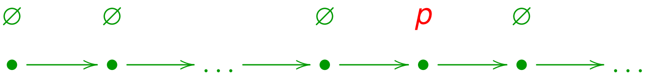
corresponding to LTL models of $\neg p \wedge \Box(\neg p \leftrightarrow \bigcirc p)$.

Some examples

- Example 2: consider TEL models of $\Diamond p$

  is like $p \vee \bigcirc p \vee \bigcirc\bigcirc p \vee \ldots$

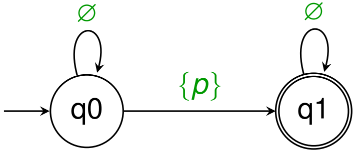
  TEL models have the form



corresponding to LTL models of $\neg p \,\mathcal{U}\, (p \wedge \bigcirc \Box \neg p)$

An example

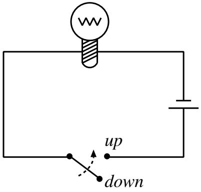
- In ASP terms, how can we represent temporal stable models? infinitely long, infinitely many



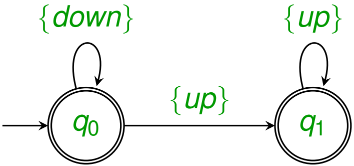
- Answer: using Büchi automata. An infinite-length word is accepted iff it visits some acceptance state infinitely often

Some examples

- Example: a lamp switch



		$up \vee down$	Initially
$\square(up \wedge \neg\bigcirc down$	$\rightarrow$	$\bigcirc up)$	Inertia
$\square(down \wedge \neg\bigcirc up$	$\rightarrow$	$\bigcirc down)$	Inertia
		$\square(up \vee \neg up)$	Choice



We never get $up \wedge down$
Once up is true, it remains so forever

Contributions . . .

In the paper you will find

- A translation of TEL(+past) into Quantified Equilibrium Logic (Kamp's translation, in fact)
- A (Tseitin-like) method to remove past operators by introducing auxiliary atoms

Kamp's translation

$$
\begin{aligned}
[\bot]_t &\stackrel{def}{=} \bot \\
[p]_t &\stackrel{def}{=} p(t), \text{ with } p \in At. \\
\neg, \wedge, \vee, \rightarrow &\stackrel{def}{=} \text{propagates to subformulas} \\
[\bigcirc\alpha]_t &\stackrel{def}{=} [\alpha]_{t+1} \\
[\alpha \; \mathcal{U} \; \beta]_t &\stackrel{def}{=} \exists x \geq t. \, ([\beta]_x \wedge \forall y \in [t, x). \, [\alpha]_y) \\
[\alpha \; \mathcal{S} \; \beta]_t &\stackrel{def}{=} \exists \, 0 \leq x \leq t. \, ([\beta]_x \wedge \forall y \in (x, t]. \, [\alpha]_y)
\end{aligned}
$$

Theorem

$[\cdot]_t$ *is sound both for THT and TEL*

Removing past operators

- Each "past" subformula χ is replaced by a new auxiliary proposition $\mathbf{L}_\chi$
- Then, we add the axioms

$$df(\chi) \stackrel{def}{=} \begin{cases} \Box(\bigcirc \mathbf{L}_\chi \leftrightarrow \varphi) \wedge (\mathbf{L}_\chi \leftrightarrow \top) & \text{if } \gamma = \ominus\varphi; \\ \Box(\bigcirc \mathbf{L}_\chi \leftrightarrow \varphi) \wedge (\mathbf{L}_\chi \leftrightarrow \bot) & \text{if } \gamma = \widehat{\ominus}\varphi; \\ \Box(\bigcirc \mathbf{L}_\chi \leftrightarrow \bigcirc\psi \vee (\bigcirc\varphi \wedge \mathbf{L}_\chi)\ ) \wedge (\mathbf{L}_\chi \leftrightarrow \psi) & \text{if } \gamma = (\varphi\ \mathcal{S}\ \psi); \\ \Box(\bigcirc \mathbf{L}_\chi \leftrightarrow \bigcirc\psi \wedge (\bigcirc\varphi \vee \mathbf{L}_\chi)\ ) \wedge (\mathbf{L}_\chi \leftrightarrow \psi) & \text{if } \gamma = (\varphi\ \mathcal{T}\ \psi). \end{cases}$$

Theorem

THT(+past) models of Γ = (pure future) THT models of translation Γ' (after ignoring auxiliary atoms).

Open topics

- Implementation on current tools
- Use for incremental ASP (clingo)
- Analysis of fundamental properties

Temporal Equilibrium Logic with Past Operators

Aguado, Cabalar, Diéguez, Pérez & Vidal

Thanks for your attention

Thank you for your participation!

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