Temporal Constraint Networks in Action: A preliminary report

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Abstract

This paper presents a study for the logical formalization of Temporal Constraint Networks (TCN), and how it can be introduced in an nonmonotonic logic for reasoning about actions and change called AL^2 [9]. The resulting logic, AL^2_{TC}, combines the representational capabilities of both formalisms, allowing temporal constraints in the conditions of causal rules that describe the domain behavior. We show how one of the main features of AL^2, called *pertinence*, provides an appropriate basis for directly relating time points not only to action occurrences, but also to relevant changes in fluents.

1 Introduction

One of the research areas inside Artificial Intelligence to which more attention has been paid is Temporal Reasoning. Despite of the great variety of approaches for representing and reasoning with time and dynamic domains, they can be classified into three main trends: *nonmonotonic temporal reasoning* – action formalisms –, *temporal modal logic* and *temporal constraints* (both qualitative and quantitative). Each trend has grown more or less independently and has been motivated by different goals, addressing a particular kind of temporal problems. Thus, for instance, quantitative temporal constraints are concerned with solving numerical relations between a given set of events that are known to have happened. On a different viewpoint, action formalisms pay more attention to the description of the system properties and how good is the way in which their dynamic behavior is finally represented. In this way, action formalisms are required to avoid a set of representational problems that usually arise when dealing with dynamic domains (the most usual are the so-called *frame*, *ramification* and *qualification* problems).

Although considerable effort has been done in mixing action formalisms and temporal modal logic [13; 1; 2], there is not much work studying the possibilities and consequences of combining action based approaches with temporal constraints (as an exception, see [12]).

In this work we present a study for the logical formalization of *Temporal Constraint Networks* (TCN) [3], and how to incorporate it inside a nonmonotonic logic for reasoning about actions and change called AL^2 [9]. In this way, we augment the expressivity, both of the action formalism and of the kind of problems solved by TCN. Besides, in future work, this will allow applying, for restricted types of action theories, the already known inference methods for temporal constraint networks.

This paper is organized as follows: in the next section we introduce L_{TC}, a logic for representing temporal constraints. In section 3, we extend L_{TC} with the classical logical connectives, what allows to represent Temporal Constraint Networks as L_{TC} theories, and even augment their representational capabilities. Section 4 describes the nonmonotonic action logic AL^2, and its extension for dealing with temporal constraints, AL^2_{TC}. In section 5, we establish a comparison to another existing related approach, which mixes temporal constraints and *Event Calculus*. Finally we draw some conclusions and present directions for future work.

2 L_{TC} Syntax and Semantics

The language of L_{TC} is formed by a finite set of *time point names*, $TP = \{t_1, ...t_n\}$, and an infinite set of nonnested binary connectors of the shape $\trianglerighteq_d$, being d an integer number. Initially, we will only define one kind of well formed formula, called *atomic*[1] *constraint*, with shape:

$$t_1 \trianglerighteq_d t_2 \tag{1}$$

where $t_1, t_2 \in TP$ and d is any integer number. Intuitively, this constraint points out that the elapsed time from t_1 to t_2 is, *at most*, d temporal units. An L_{TC} *theory* will be a set of these atomic constraints.

The semantics of L_{TC} relies on assigning an integer value to each time point name.

Definition 1 (Interpretation) An *interpretation* M is a mapping from the set of time points TP to the integer numbers $M : TP \longrightarrow \mathbf{Z}$. □

[1] The name *atomic* refers to the fact that we will not introduce any logical connective or quantifier at this point. We could alternatively see each constraint as a ground atom of a ternary predicate $\trianglerighteq (t_1, t_2, d)$.

Definition 2 (Satisfiability) We say that an interpretation M *satisfies* a constraint $t_1 \trianglerighteq_d t_2$, denoted as $M \models t_1 \trianglerighteq_d t_2$, iff $M(t_2) - M(t_1) \leq d$. □

As we can see, a constraint $t_1 \trianglerighteq_d t_2$, with $d > 0$, fixes a maximum value for the elapsed time between t_1 and t_2. Notice that constraint distances can be negative. In such a case, we would have that:

$$M \models t_1 \trianglerighteq_{-d} t_2$$

where $d > 0$, is equivalent to:

$$M(t_1) - M(t_2) \geq d$$

that is, d is a minimum for the converse distance time from t_2 to t_1.

As usual, we say that an interpretation M is a *model* of a theory T iff it satisfies all the constraints in T. In the same way, a theory T is said to *entail* a constraint F iff all the models of T are models of F.

An interesting property verified by L_{TC}, is the transitivity of constraints:

Property 1 (Transitivity) *Any model M of the constraints $t_1 \trianglerighteq_d t_2$ and $t_2 \trianglerighteq_e t_3$, is also a model of $t_1 \trianglerighteq_{d+e} t_3$.*

Proof Since M is a model of the first two constraints, it must satisfy:

$$M(t_2) \leq M(t_1) + d \quad (2)$$

$$M(t_3) \leq M(t_2) + e \quad (3)$$

Adding e to both sides of (2) we get:

$$M(t_2) + e \leq M(t_1) + d + e \quad (4)$$

Applying transitivity (of the integer ordering relation) to (3) and (4) we obtain $M(t_3) \leq M(t_1) + d + e$, which is equivalent to say that M satisfies $t_1 \trianglerighteq_{d+e} t_3$. □

In order to relate this logical reformulation of Temporal Constraints to its original definitions [3], we will establish some direct correspondences with respect to the "classical" terminology. An L_{TC} theory T corresponds to what is called a *distance graph* where each constraint $t_1 \trianglerighteq_d t_2$ in T is represented as an arc from t_1 to t_2 with distance d. A model of a theory T is also known as *solution*. Given a theory T, we say that a value v is *feasible* for a time point t iff there exists a model M of T such that $M(t) = v$. The set of feasible values of t receives the name of *minimal domain*.

A typical problem in Temporal Constraints is to check the *consistency* of a graph, that is, checking the existence of a solution or, in L_{TC} terms, the existence of a model. We will show that the traditional consistency checking method introduced in [3] is also sound and complete with respect to L_{TC}. In short, the method consists in looking for negative cycles inside the distance graph. When translated into L_{TC} terms, it will consist in a syntactic test that looks for a particular kind of subtheory (the negative cycle) inside a theory T.

Let us introduce first some definitions and preliminary results. Given a theory T, we define a *path* between t and t', as a subtheory of T consisting of a sequence of constraints like:

$$\{t \trianglerighteq_{d_1} t_1 \ , \ t_1 \trianglerighteq_{d_2} t_2 \ , \ldots , \ t_{n-1} \trianglerighteq_{d_n} t'\}$$

By $Paths(t, t')$ we denote the set of paths in T between t and t'. A *cycle* is a non-empty path from t to t. Given a path p, we define its *distance*, $dist(p)$, as the summation of all the d_i in p. The notation $dist(Paths(t, t'))$ is an abbreviation of the set $\{dist(p), p \in Paths(t, t')\}$.

Theorem 1 *An L_{TC} theory T is inconsistent iff it contains a cycle with negative distance.*

Proof
"$\Longleftarrow$" (Soundness)

Let p be a negative cycle in T. By iterative application of transitivity to the constraints in p, any model of T should satisfy $t \trianglerighteq_{dist(p)} t$, that is, $M(t) - M(t) \leq dist(p)$, which is not possible, since $dist(p) < 0$.

"$\Longrightarrow$" (Completeness)

We will prove the result by *reductio ad absurdum*. To this aim, we will assume that there are no negative cycles in T and try to find a contradiction. Let TP_0 be the set of non-dependent time points:

$$TP_0 = \{t \in TP, \text{ s.t. there is no } (t' \trianglerighteq_d t) \in T\}$$

We will first prove the result for $TP_0 \neq \emptyset$. In such a case, we can construct a mapping M from TP to the integer numbers where:

i) $M(t) = 0$ for any $t \in TP_0$

ii) $M(t) = min\{dist(Paths(t', t)), \ t' \in TP_0\}$ for any $t \notin TP_0$

that is, we can assign to $M(t)$ the minimum distance of the paths from time points t' in TP_0. Note that this assignment is possible because all the cycles are of non-negative distance. Otherwise, the (possibly infinite) set $Paths(t', t)$, could have no defined minimum distance, as negative cycles could allow infinitely decreasing the distance of some cyclic path.

We prove now that M satisfies any arbitrary constraint $c = (t_1 \trianglerighteq_d t_2)$ in T. By construction, $M(t_1)$ is the distance of the shortest path p_1 to t_1 from any time points in TP_0; $M(t_1) = dist(p_1)$. Let us consider the path p_2 that results from appending c to p_1. The distance of p_2 is $dist(p_1) + d$, i.e., $M(t_1) + d$. Since p_2 constitutes a path from some $t' \in TP_0$ to t_2, by construction again, $M(t_2) \leq dist(p_2)$, that is $M(t_2) \leq M(t_1) + d$, and so, the constraint c is satisfied. Then M is a model of T, which contradicts the initial hypothesis of inconsistency of T.

For the case of $TP_0 = \emptyset$, the same reasoning can be applied. It suffices with identifying an alternative set of reference points TP'_0 (to be assigned to 0) so that, for any t, there exists $t' \in TP_0$, such that $Paths(t', t) \neq \emptyset$. Such set can be constructed by choosing one arbitrary point from each maximal *strongly connected component* (SCC) that does not depend on another SCC. □

3 General L_{TC} and Temporal Constraint Networks

As we have seen in the previous section, an L_{TC} theory can actually be seen as a set of simple atomic expressions, which is a strong expressive limitation. For instance, we practically cannot introduce any kind of disjunctive knowledge. The unique way of expressing uncertainty is by imposing minimum and maximum bounds to temporal differences between time points. Because of this limitation, these kind of temporal problems have received the name of *Simple Temporal Problems* (STP).

In [3], a more expressive representation called *Temporal Constraint Networks* (TCN) was introduced. A TCN is a directed graph where nodes are time point names and each arc is labeled by a set of intervals. Arc (t_1, t_2) labeled by an interval $[a, b]$ represents that the elapsed time from t_1 to t_2 must be contained in interval $[a, b]$. If an arc is labeled by several intervals, it is understood that at least one of them must be satisfied.

When each arc is labeled by a single interval we have an STP and it can be directly represented in L_{TC} by replacing each arc:

$$t_1 \xrightarrow{[a,b]} t_2 \tag{5}$$

by the pair of atomic constraints:

$$t_1 \trianglerighteq_b t_2$$
$$t_2 \trianglerighteq_{-a} t_1$$

Representing general TCN, however, requires additional expressivity. To this aim, we propose extending L_{TC} for dealing with usual propositional connectives. Thus, we extend the syntax allowing any propositional combination ($\wedge$,$\vee$ and $\neg$) of atomic constraints. The satisfiability of formulas would be extended by usual truth recursion:

Definition 3 (Satisfiability) We redefine the satisfiability relation in the following way:

$M \models t_1 \trianglerighteq_d t_2$	iff	$M(t_2) - M(t_1) \leq d$
$M \models \phi \wedge \psi$	iff	$M \models \phi$ and $M \models \psi$
$M \models \phi \vee \psi$	iff	$M \models \phi$ or $M \models \psi$
$M \models \neg\phi$	iff	not $M \models \phi$

where $t_1 \trianglerighteq_d t_2$ is an atomic constraint and ϕ, ψ are general propositional constraints. □

This natural extension of L_{TC} directly allows representing general TCN. Each arc (t_1, t_2) labeled with intervals $[a_1, b_1], \ldots, [a_n, b_n]$, would be represented as the formula:

$$\bigvee_{i \in [1,n]} (t_1 \trianglerighteq_{b_i} t_2 \wedge t_2 \trianglerighteq_{-a_i} t_1)$$

which expresses in a clearer way the disjunctive nature of general TCN constraints.

The introduction of propositional connectives into L_{TC} provides, in fact, a greater expressivity. Let us think, for instance, in the simple introduction of negation for atomic constraints. We can easily see that $M \models \neg(t_1 \trianglerighteq_d t_2)$ iff $M(t_2) - M(t_1) > d$. In other words, we may introduce strict inequalities that, otherwise, should have to be introduced by defining new operators (some kind of strict $t_2 \triangleright_d t_1$).

Another example of the expressive power of the extended L_{TC} is the possibility of expressing logical properties in a syntactic way. For instance, the transitivity property corresponds now to the schemata:

$$t_1 \trianglerighteq_d t_2 \wedge t_2 \trianglerighteq_e t_3 \supset t_1 \trianglerighteq_{d+e} t_3$$

where '$\supset$' represents material implication ($\phi \supset \psi$ is, as always, defined as $\neg\phi \vee \psi$).

Finally, the full combination of propositional operators could allow representing problems which cannot be represented by a TCN. Think for instance in the example theory:

$$T = \{(t_1 \trianglerighteq_0 t_2 \wedge t_2 \trianglerighteq_0 t_1 \wedge t_3 \trianglerighteq_0 t_4 \wedge t_4 \trianglerighteq_0 t_3) \vee$$
$$(t_1 \trianglerighteq_0 t_3 \wedge t_3 \trianglerighteq_0 t_1 \wedge t_2 \trianglerighteq_0 t_4 \wedge t_4 \trianglerighteq_0 t_2)$$

The models of T are those assignments M in which either $M(t_1) = M(t_2)$ and $M(t_3) = M(t_4)$, or $M(t_1) = M(t_3)$ and $M(t_2) = M(t_4)$. These kind of expressions cannot be translated into a TCN, since the disjunction affects to the whole structure of the network, and not only to the possible combinations of intervals for a fixed set of arcs.

4 AL^2_{TC}: an action formalism with temporal constraints

In this section we study how to extend an actions formalism, AL^2, for dealing with temporal constraints like those in L_{TC}.

First, we recall the basic definitions of AL^2 and then we extend them for dealing with temporal constraints.

4.1 AL^2 syntax and semantics

In [9], a narrative-based causal actions logic, AL^2, was introduced. Its syntax depends on a fixed parameter n which delimits the length of the narratives, and on two disjoint sets: the *action names*, $\mathcal{A}$, and the *fluent names*, $\mathcal{F}$. We will use the letter q to denote any action or fluent name, and letters f and a to denote fluents and actions respectively.

An *atom* has the shape q_i where $i \in [0, n]$ is a non-negative integer, pointing out a situation along the narrative. By $Atoms(\mathcal{A}, \mathcal{F}, n)$ we denote the set of all possible atoms. We call a *literal* to any atom q_i or its negation $\neg q_i$. General AL^2 formulas may be of two possible types:

i) *non-causal* formulas: defined as combinations of atoms with the classical propositional connectives ($\wedge$, $\vee$,$\neg$), plus a new type of atomic construction of the shape $(\texttt{prev}\ q)_i$.

ii) *causal rules*: with shape $F \Leftarrow \phi$ where F is a literal for some f_i and ϕ is a non-causal formula such that, for any subindex j occurring in ϕ, $j \leq i$.

The restrictions imposed on causal rules intuitively say that actions are never "concluded" as a result of a rule (we will be only interested in concluding fluent literals) and that the condition of the rule must refer to the same or to previous situations, with respect to the situation of the conclusion.

The semantics of AL^2 is an action-based extension of a nonmonotonic double-valued logic called L^2, which has as main feature the introduction of an *extra valuation* of formulas, called *pertinence*, parallel to the classical truth valuation. This additional valuation points out if a formula has been *affected by change*, i.e., if it has been caused to take a new value.

Informally speaking, a formula will be pertinent if either we have explicitly asserted it to be so, or it is affected by other pertinent expressions. If we apply this general idea for action theories, a fluent will be said to be pertinent when it is directly or indirectly affected by an action.

Formally, an interpretation in AL^2 will assign a pair of values to each formula. Apart from the usual truth value (true or false, $\{\mathtt{t},\mathtt{f}\}$), a formula will also be assigned a *pertinence* value: "pertinent" or "non-pertinent", denoted respectively as $\mathtt{p}$ and $\mathtt{n}$. An AL^2 *interpretation* I is a mapping $I : Atoms(\mathcal{A},\mathcal{F},n) \longrightarrow \{\mathtt{t},\mathtt{f}\} \times \{\mathtt{p},\mathtt{n}\}$ assigning truth and pertinence to each atom. In this way, though the underlying concept remains as a dual valuation, we will technically handle a multivalued logic with the four combinations $\{\mathtt{tp},\mathtt{tn},\mathtt{fp},\mathtt{fn}\}$. An interpretation I may be alternatively described by a pair $\langle S,P\rangle$ of sets of atoms for representing truth and pertinence respectively. In this way, given $\langle S,P\rangle$, the interpretation of an atom q_i is simply established as:

$$\begin{array}{ll} I(q_i) \in \{\mathtt{tp},\mathtt{tn}\} & \text{iff } q_i \in S \\ I(q_i) \in \{\mathtt{tp},\mathtt{fp}\} & \text{iff } q_i \in P \end{array}$$

that is, q_i is true iff it belongs to S and it is pertinent iff it belongs to P.

There are, however, two particular restrictions depending on the type of the atom q_i. First, when the atom is placed in the initial situation, $i = 0$, it will always be non-pertinent. This means that there does not exist any $q_0 \in P$. This restriction is imposed for fixing the initial situation with respect only to truth, avoiding application of rules (due to pertinence) at the initial state. Second, when the atom is an action, a_i, it will be assigned to be pertinent iff it is assigned to be true. In other words, interpretation $\langle S,P\rangle$ must satisfy: $a_i \in P$ iff $a_i \in S$. This restriction intuitively points out that an action is pertinent iff it has occurred. In combination with the first restriction, it also implies that no action may occur at the initial situation.

Interpretations are extended in order to assign a truth and a pertinence value to each possible formula, following the rules in Figure 1.

Atomic expressions of shape $(\mathtt{prev}\ q)_i$ are used to simply check the previous truth value of some symbol q without regarding its pertinence. For this reason, a $\mathtt{prev}$ expression is never assigned to be pertinent. For propositional formulas, the truth interpretation is the usual one, whereas pertinence is valuated independently. A propositional formula is pertinent iff at least one of the atomic expressions occurring in the formula is pertinent. Finally, causal formulas are the only way of relating truth and pertinence. A causal formula will be true when: if its condition is true and pertinent, then its consequent is also true and pertinent.

$$I((\mathtt{prev}\ q)_i) = \begin{cases} \mathtt{tn} & \text{if } I(q_{i-1}) \in \{\mathtt{tp},\mathtt{tn}\} \\ \mathtt{fn} & \text{if } I(q_{i-1}) \in \{\mathtt{fp},\mathtt{fn}\} \end{cases}$$

$$I(\neg\phi) = \begin{cases} \mathtt{t}\alpha & \text{if } I(\phi) = \mathtt{f}\alpha \\ \mathtt{f}\alpha & \text{if } I(\phi) = \mathtt{t}\alpha \end{cases}$$

$$I(\psi \Leftarrow \phi) = \begin{cases} \mathtt{tn} & \text{if } I(\phi) \neq \mathtt{tp} \\ \mathtt{fn} & \text{if } I(\phi) = \mathtt{tp} \text{ and } I(\psi) \neq \mathtt{tp} \\ \mathtt{tp} & \text{otherwise} \end{cases}$$

$I(\phi \wedge \psi)$	fn	fp	tn	tp
fn	fn	fp	fn	fp
fp	fp	fp	fp	fp
tn	fn	fp	tn	tp
tp	fp	fp	tp	tp

$I(\phi \vee \psi)$	fn	fp	tn	tp
fn	fn	fp	tn	tp
fp	fp	fp	tp	tp
tn	tn	tp	tn	tp
tp	tp	tp	tp	tp

Figure 1: Interpretation of a formula.

Typically, an AL^2 domain description will consist of two sets of formulas $\langle B,O\rangle$: the set B called *background knowledge*, which contains causal rules, and the set O of *observations*, containing non-causal formulas. We will be interested in causal rules that are satisfied for any situation in the narrative. To this aim, we use an abbreviation allowing causal rules with variable subindices. For instance, a rule like:

$$f_i \Leftarrow a_i \wedge g_{i-1}$$

actually represents the set of rules:

$$\begin{array}{rcl} f_1 & \Leftarrow & a_1 \wedge g_0 \\ f_2 & \Leftarrow & a_2 \wedge g_1 \\ & \vdots & \\ f_n & \Leftarrow & a_n \wedge g_{n-1} \end{array}$$

When defining the concept of model in a multi-valued logic, we will need to select a value as the one that points out that a formula is satisfied. By default, in AL^2 the selected value is any one from $\{\mathtt{tp},\mathtt{tn}\}$, that is, any *true* value, without regarding pertinence. Thus, given a theory T in AL^2, an interpretation I is said to be a *model* iff it assigns true to each formula $\phi \in T$.

The set of models of T induces a monotonic entailment relation that is not enough expressive for solving the typical representational problems in action domains. To be precise, it makes impossible to represent the default rule of *inertia* which asserts: "if there is no evidence on the contrary, everything remains unchanged." The absence of an inertia default rule directly leads to the well-known *frame* problem [8; 14], forcing to represent not only when things happen to change but also adding formulas to represent when things *do not change*. On the other hand, with respect to the intuitive meaning of pertinence, it seems natural to require that when nothing points out if a fluent is pertinent in a given theory T, it should be assumed not to be so.

All these reasons lead to the introduction of a models selection process which in AL^2 solves simultaneously the already mentioned frame problem and the causal behavior of the conditional expressions $F \Leftarrow \phi$ (which also solves the so-called *ramification* problem [16; 6; 7; 4; 15]). This models selection has two steps: first, we minimize pertinence among models (*selected* models) and, second, we require all the models to satisfy an *a posteriori* condition related to inertia (*causal* models).

Definition 4 ($\leq_{\mathtt{p}}$) Given two AL^2 interpretations $I = \langle S, P\rangle$ and $I' = \langle S', P'\rangle$, we say that I is lower than I', $I \leq_{\mathtt{p}} I'$, iff $S = S'$ and $P \subseteq P'$. □

Definition 5 (AL^2 selected model) A model $I = \langle S, P\rangle$ of a theory T is said to be a *selected model* iff it is a minimal model, with respect to the $\leq_{\mathtt{p}}$ ordering relation. □

Definition 6 (Causal model) A selected model $I = \langle S, P\rangle$ of a theory T is said to be *causal* iff satisfies the condition: for any $i > 0$, and any f_i, if $f_i \notin P$ then ($f_i \in S$ iff $f_{i-1} \in S$). □

Intuitively, selected models impose the condition that things are nonpertinent by default, whereas causal models add to this requirement that any fluent nonpertinent should maintain its previous truth value. The entailment relation induced by causal models, $\models_c$, is now nonmonotonic. Let us see an example:

Example 1 (Yale Shooting scenario) We have a gun for killing a turkey. To this aim, we may perform an action *load* that makes the gun become *loaded*, and an action *shoot* that kills the turkey if the gun was loaded. Initially, the gun is loaded, the turkey is alive and we perform action *shoot*. □

Let $\mathcal{A} = \{load, shoot\}, \mathcal{F} = \{loaded, alive\}$ and $n = 1$. The domain description for the example could be simply formulated in AL^2 by the theory $T = \langle B, O\rangle$, where B contains the causal rules:

$$loaded_i \Leftarrow load_i \tag{6}$$

$$\neg alive_i \Leftarrow shoot_i \wedge (\mathtt{prev}\ loaded)_i \tag{7}$$

and O is the set:

$$\{alive_0, loaded_0, shoot_1, \neg load_1\}$$

Theorem 2 *The unique causal model of T is $I = \langle S, P\rangle$ where:*

$$S = \{alive_0, loaded_0, loaded_1, shoot_1\}$$
$$P = \{shoot_1, alive_1\}$$

Proof
When we consider the AL^2 models, we can see that, since action *shoot* is performed in situation 1, the antecedent of (7) is true for $i = 1$ and so, any model I must make $I(alive_1) = \mathtt{fp}$. However, the value of the fluent $loaded_1$ is free, and so we will have four models, one for each possible interpretation of this fluent. When considering the selected models, the pertinence minimization eliminates the models in which $loaded_1$ is pertinent, and so only two models remain: I, where $I(loaded) = \mathtt{tn}$ and I', where $I'(loaded) = \mathtt{fn}$. Finally, this last model is ruled out when considering causal models, since it makes $loaded_1$ not pertinent, whereas it has changed from true in situation 0 to false in situation 1. So, we obtain that the unique causal model is I. □

Notice that our background knowledge does not specify what happens to *loaded* when we *shoot*, and so, inertia takes charge of keeping *loaded* true, as in the initial situation. Let us see what happens if we add a rule stating that the shot unloads the gun:

$$\neg loaded_i \Leftarrow shoot_i \wedge (\mathtt{prev}\ loaded)_i \tag{8}$$

Now, this new rule is also applicable at situation 1, fixing $I(loaded) = \mathtt{fp}$. In fact, this leads to a unique AL^2 model which is trivially a selected model, and can be easily seen to be causal.

This example shows the nonmonotonic property of the $\models_c$ relation, as we had that $T \models_c loaded_1$, whereas, adding the new rule, $T \cup (8) \not\models_c loaded_1$ (in fact, $T \cup (8) \models_c \neg loaded_1$).

4.2 Adding temporal constraints to AL^2

In this section, we extend the formalization of AL^2 into a new logic called AL^2_{TC} which incorporates temporal constraints. The syntax of AL^2 is extended by introducing new atomic expressions (the already seen atomic constraints) and combining them freely in any noncausal formula.

The intended utility of the temporal constraints will be to impose or test conditions on the elapsed *number of situations* that have occurred between two given events. However, this immediately gives rise to a question: what is considered to be an *event* in an action-based formalism? or, in other words, which will be the set of time points, *TP* for the atomic constraints?

A first natural answer to this question is thinking on action occurrences as events, and on the situations in which they occur as time points. However, in AL^2 it is possible to go further, and consider also as an event the moment in which any atom, action or fluent, becomes pertinent. This results in a richer expressivity, allowing temporal constraints to relate instants that depend on fluents.

Let us introduce the formal definitions. Given a set of actions $\mathcal{A}$, a set of fluents $\mathcal{F}$ and the fixed length n, we define the set of *time point names* as:

$$TP(A,F,n) = \{t(q)_i, \text{ for any } q \in \mathcal{A} \cup \mathcal{F} \text{ and } i \in [0,n]\}$$

Notice that time point names have also a subindex, that is, they depend on the situation in which they are placed. The intuitive meaning of $t(q)_i$ is that it represents, at situation i, the number of the latest situation, $j \leq i$, in which q became pertinent.

The syntax of AL^2_{TC} is the same one of AL^2 but allowing a new type of atomic expression called *atomic constraint* with shape:

$$t(q)_i \trianglerighteq_d t(q')_i \tag{9}$$

Notice that the set of events we are handling introduces a new feature not considered in traditional temporal constraint problems: an event may have not occurred. As we will see, we will consider that the satisfaction of any temporal constraint like (9) will imply that both q and q' have actually occurred in the past. For this reason, we will call *strong* to these kind of constraints and, for the expressivity sake, we will define the dual *weak* constraints:

$$t(q)_i \succeq_d t(q')_i \tag{10}$$

that will allow that q or q' have not occurred.

The semantics of AL^2_{TC} is defined in the following way. An AL^2_{TC} interpretation I is defined as a triple $I = \langle S, P, M \rangle$, where S and P have the same meaning than in AL^2 and M is a mapping from $TP(A,F,n)$ to the nonnegative integer numbers $[0,n]$, satisfying $M(t(q)_i) \leq i$, for any q_i. An assingment $M(t(q)_i) = 0$ will point out that q_i has never become pertinent. Interpretation formulas in AL^2_{TC} just introduce, with respect to rules in Figure 1, the interpretation of these new atomic constraints:

$$I(t(q)_i \trianglerighteq_d t(q')_i) = \begin{cases} \mathtt{tn} & \text{if } M(t(q)_i) \neq 0 \text{ and} \\ & M(t(q')_i) \neq 0 \text{ and} \\ & M(t(q')_i) - M(t(q)_i) \leq d \\ \mathtt{fn} & \text{otherwise} \end{cases}$$

$$I(t(q)_i \succeq_d t(q')_i) = \begin{cases} \mathtt{tn} & \text{if } M(t(q)_i) = 0 \text{ or} \\ & M(t(q')_i) = 0 \text{ or} \\ & M(t(q')_i) - M(t(q)_i) \leq d \\ \mathtt{fn} & \text{otherwise} \end{cases}$$

As we can see, temporal constraints are never assigned a pertinence value (they behave in a similar way to the `prev` operator seen before). Besides, strong constraints require both compared instants to be different from 0, whereas weak constraints will be directly true whenever any of those instants happen to be 0.

We directly recall the definition of *model* in AL^2, using now this extended definition of interpretation. As for *selected* models, we only need to redefine the ordering relation in the following way:

Definition 7 ($\leq_{\mathtt{P}}$) Given two AL^2_{TC} interpretations $I = \langle S, P, M \rangle$ and $I' = \langle S', P', M' \rangle$, we say that I is lower than I', written $I \leq_{\mathtt{P}} I'$, iff $S = S'$, $M = M'$ and $P \subseteq P'$. □

That is, in order to minimize pertinence, we fix not only the truth S, but also the time points assignment M.

Definition 8 (Causal model) A selected model $I = \langle S, P, M \rangle$ of a theory T is said to be *causal* iff it satisfies the conditions:

1. For any $f_i \notin P$, $f_i \in S$ iff $f_{i-1} \in S$
2. For any $M(t(q)_i) = 0$, there is no $q_j \in P$, $j \leq i$
3. For any $M(t(q)_i) \neq 0$,
   $M(t(q)_i) = max\{j \leq i \text{ s.t. } q_j \in P\}$

□

The two new conditions fix the meaning of time points with respect to the pertinences occurring in the selected model. For time points $t(q)_i$ assigned to 0, we require that q has never been pertinent until situation i. For time points different from 0, the assigned value must coincide with last recent situation j before i in which q became pertinent.

Notice that, until this step of models selection, we had not imposed any relation between the assigned number to a time point $t(q)_i$ and the information about q_i contained in the model. This means that, for AL^2_{TC} models and selected models, the assigned numbers to time points are completely arbitrary with respect to pertinence of fluents and actions.

We will illustrate how temporal constraints can be mixed with an usual action theory by an elaboration of example 1.

Example 2 (Spoiling Yale Shooting Scenario) Assume now that a shot only kills the turkey if we performed a load at most two situations before (otherwise, the gunpowder in the bullet spoils). Initially the turkey is alive and the gun unloaded. We load the gun and after two situations without performing any action, we shoot. □

The domain description T_{spoil} for this scenario consists of the set B of causal rules:

$$\begin{aligned} loaded_i &\Leftarrow load_i \qquad (11) \\ \neg alive_i &\Leftarrow shoot \wedge (\mathtt{prev}\ loaded)_i \\ &\qquad \wedge (t(load)_i \trianglerighteq_2 t(shoot)_i) \qquad (12) \\ \neg loaded_i &\Leftarrow shoot \wedge (\mathtt{prev}\ loaded)_i \qquad (13) \end{aligned}$$

and the set O of observations:

$$\begin{aligned} \{&alive_0, \neg loaded_0, \\ &\neg shoot_1, load_1, \\ &\neg shoot_2, \neg load_2, \\ &\neg shoot_3, \neg load_3, \\ &shoot_4, \neg load_4\} \end{aligned}$$

The intended result is that the turkey should be eventually alive, since the temporal constraint for killing it does not hold.

Theorem 3 *The unique causal model of T_{spoil} is $I = \langle S, P, M\rangle$ where:*

$$S = \{alive_0, load_1, loaded_1, alive_1, loaded_2, alive_2, loaded_3, alive_3, shoot_4, alive_4\}$$
$$P = \{load_1, loaded_1, shoot_4, loaded_4\}$$

and M is defined by the following table:

$M\backslash i$	0	1	2	3	4
$t(alive)_i$	0	0	0	0	0
$t(loaded)_i$	0	1	1	1	4
$t(shoot)_i$	0	0	0	0	4
$t(load)_i$	0	1	1	1	1

Proof

We first show that I is causal model of T_{spoil}. Following AL^2_{TC} satisfiability of formulas, it can be easily seen that I is model [2] of T_{spoil}. With respect to pertinence minimization, we can observe that, if we fix S and M, we have models for any possible pertinence combination of *alive*, in situations $i \in [1, 4]$, and *loaded*, in situations $i \in [2, 3]$. The unique minimal model is the one where all these atoms are nonpertinent, and this is precisely our model I. Finally, I is also a causal model, since it satisfies the conditions in definition 8.

Let us see now if I is the unique causal model of T_{spoil}. If we consider the fixed facts due to observations, the only rule whose antecedent is always applicable is (11) in situation 1, and so, any model should assign $M(loaded_1) = \mathtt{tp}$. The rest of rules are always not applicable, excepting (12) and (13) in situation 4. We will show that no causal model could make false their conditions by making $loaded_3$ false. Without regarding the truth value of $loaded_2$, the minimization of pertinence would select models in which $loaded_2$ and $loaded_3$ are nonpertinent. However, such models cannot be causal, because they assign a change in the truth value to either $loaded_2$ or $loaded_3$. Therefore, we assume instead that *loaded* is true in situations 2 and 3. Now rule (13) is always applicable, and so any model must assign $M(loaded_4) = \mathtt{fn}$.

Taking a look to the temporal constraint, we can easily see that, having the occurrence of actions fixed by the observations, there is no $M' \neq M$ that could eventually satisfy conditions in definition 8. So, M is fixed, and condition of rule (12) becomes false. Following the same reasoning for *alive* that the one in the previous explanation for *loaded*, we can conclude that *alive* must remain true and nonpertinent, what corresponds now to a unique causal model that is precisely I. □

It can be similarly checked that, if we load the gun, we spend a situation without doing anything and then we shoot the gun, then, the only AL^2_{TC} model is $I' = \langle S, P, M\rangle$ where:

[2]Focusing, for instance, on rule (12) at situation 4, the temporal constraint is not satisfied – $(M(t(load)_4) - M(t(shoot)_4) = 3$ which is greater than 2), and so the whole rule is true (and not pertinent).

$$S = \{alive_0, loaded_1, alive_1, loaded_2, alive_2\}$$
$$P = \{load_1, loaded_1, shoot_3, loaded_3, alive_3\}$$

and M is defined by the following table:

$M\backslash i$	0	1	2	3
$t(alive)_i$	0	0	0	3
$t(loaded)_i$	0	1	1	3
$t(shoot)_i$	0	0	0	3
$t(load)_i$	0	1	1	1

5 Related work

Few bibliography can be found relating temporal constraints with action-based formalisms. The most relevant existing approach is perhaps the language `HOT` introduced in [12], which mixes temporal constraints with a well known formalism for reasoning about change: the *Event Calculus* [5]. In this section, we briefly outline the main similarities and differences between `HOT` and AL^2_{TC}.

Language `HOT` handles three basic predicates: *Holds*, *Occurs* and *Time*. The first two are usual in Event Calculus: $Holds(\phi, E_1, E_2)$ allows asserting that a given fact ϕ permanently holds between two given events E_1, E_2, and $Occurs(E)$ allows asserting that a given event E has actually occurred. A new function $Time(E)$ is used for associating to each event a real number that represents the instant in which the event occurred. Besides classical formulae combining *Holds* and *Occurs* with propositional operators, `HOT` allows defining both qualitative and quantitative constraints, that are atomic expressions relating the times of the events. We will focus the attention on quantitative constraints, which have the shape:

$$Time(E) - Time(E') \in I_1 \cup I_2 \cup \ldots I_k \qquad (14)$$

where I_i are intervals over real numbers. In the sight of these expressions, several coincidences arise with respect to AL^2_{TC}. For instance, time points $Time(E)$ in `HOT` associate a real number to an event, whereas in AL^2_{TC} we have $t(q)_i$, assigning a nonnegative integer that refers to the situation in which the event occurred, rather than to its time. Notice that, the role of events in AL^2_{TC} is played by facts like "the last occurrence of q pertinent." In this way, it is possible to express also temporal constraints between fluents, using pertinence not only for implementing causality, but also as a valuable information that allows fixing the instants in which fluents were *affected* by change.

Another common feature is the possible non-occurrence of events. As we had said, traditional TCN problems consider that all the events associated to time points have actually occurred. So, the need of possible non-occurrence of events seems to be a common feature to any action and change formalism. In `HOT` the selected choice is, in order to make true a constraint, to satisfy not only the corresponding numerical relation, but also the occurrence of the related events. In other words, `HOT` constraints directly correspond to what we have called *strong* temporal constraints in AL^2_{TC}.

Despite of these similarities, particularly related to the way in which time points are defined, there are some major differences between the orientation of HOT and AL^2_{TC}. The most important one is that HOT has not considered the typical representation problems of reasoning about actions and change (specially the frame and ramification problems). This becomes evident when we take a look to approaches based on Event Calculus, which traditionally define two additional predicates, *Initiates* and *Terminates* that need to be minimized in order to cope with the default rule of inertia. In HOT , this nonmonotonic behavior of Event Calculus has been omitted. Our approach means, in this sense, a significative advance, since it integrates temporal constraints with a nonmonotonic framework that solves simultaneously the frame and the ramification problems[3], by incorporating causality to the rules in the background knowledge.

6 Conclusions

We have presented a logical formalization for Temporal Constraint Networks which can be integrated into the nonmonotonic logic AL^2, for reasoning about actions and change. The resulting formalism, AL^2_{TC} combines interesting representational features from these two different temporal approaches.

From the AL^2 point of view, the new features provide a first natural step towards the introduction of continuity. Our approach differs, in this sense, from the usual treatment given to continuity in action based approaches [11; 10], which handles fluents with a real domain that *evolve continuously*, while the number of situations or the distances among them are kept constant. As opposed to this, our approach begins the introduction of continuity by defining the distances between occurrences of events to be non-constant. In fact, for the sake of simplicity, we have first studied the simple case in which we represent the number of situations elapsed between events, instead of handling continuous time distances.

From the temporal constraint networks point of view, the features provided by an action formalism mean a significative increase in representational power. We have avoided two strong restrictions that are implicit assumptions in typical TCN problems: (1) the events are always known to have occurred and cannot repeatedly occur and (2) each event actually contains all the information about the system state (there is no possibility to represent the different properties that are satisfied by the system at a given instant). Besides, this is done maintaining the solution to the frame and ramification problems, already present in AL^2.

As future work, we will study how the existing TCN techniques can be used for improving inference in particular kinds of AL^2_{TC} theories. For instance, we could think about theories in which the only temporal constraints are used in rules like:

$$false \Leftarrow \neg(t(load)_i \succeq_3 t(shoot)_i)$$
$$false \Leftarrow \neg(t(shoot)_i \succeq_{-2} t(load)_i)$$

These constraints are used, in fact, for limiting the instants in which actions can be performed (we can shoot only after 2 situations and before 3 once we have loaded). The interesting point is that constraints of this shape directly correspond to simple temporal problems, and so we could directly apply the standard TCN methods for consistency checking or finding a feasible set of instants.

[3] Notice that, for instance, the study on how to solve ramification problem for the Event Calculus is still a topic of current research [15].

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References

[1] Matteo Baldoni, Laura Giordano, Alberto Martelli, and Viviana Patti. An abductive proof procedure for reasoning about actions in modal logic programming. In *Proc. of Non-Monotonic Extensions on Logic Programming (NMELP'96)*, 1996.

[2] Marcos A. Castilho, Olivier Gasquet, and Andreas Herzig. Formalizing action and change in modal logic I: the frame problem. *Journal of Logic and Computation*, to appear.

[3] R. Detcher, I. Meiri, and J. Pearl. Temporal constraint networks. *Artificial Intelligence Journal*, 49:61–95, 1991.

[4] J. Gustafsson and P. Doherty. Embracing occlusion in specifying the indirect effects of actions. In *Proceedings of the 5th Intl. Conference on Principles of Knowledge Representation and Reasoning*, 1996.

[5] R. Kowalski and M. Sergot. A logic-based calculus of events. *New Generation Computing*, 4:67–95, 1986.

[6] Fangzhen Lin. Embracing causality in specifying the indirect effects of actions. In C. S. Mellish, editor, *Proceedings of the International Joint Conference on Artificial Intelligence (IJCAI)*, Montreal, Canada, August 1995. Morgan Kaufmann.

[7] Norman McCain and Hudson Turner. A causal theory of ramifications and qualifications. In C. S. Mellish, editor, *Proceedings of the International Joint Conference on Artificial Intelligence (IJCAI)*, pages 1978–1984, Montreal, Canada, August 1995. Morgan Kaufmann.

[8] J. McCarthy and P. Hayes. Some philosophical problems from the standpoint of artificial intelligence. *Machine Intelligence Journal*, 4:463–512, 1969.

[9] R.P. Otero and P. Cabalar. Pertinence and causality. In *Proceedings of the 3rd Workshop*

on Nonmonotonic Reasoning, Action, and Change (NRAC),IJCAI'99, Stockholm, August 1999.

[10] E. Sandewall. Combining logic and differential equations for describing real world systems. In H. Levesque R. Brachman and R. Reiter, editors, *Proceedings of the First International Conference on Principles of Knowledge Representation and Reasoning*, pages 412–420. Morgan Kaufmann, 1989.

[11] E. Sandewall. Filter preferential entailment for the logic of action in almost continuous worlds. In C. S. Mellish, editor, *Proceedings of the International Joint Conference on Artificial Intelligence (IJCAI)*, pages 894–899. Morgan Kaufmann, 1989.

[12] E. Schwalb, K. Kask, and R. Decther. Temporal reasoning with constraints on fluents and events. In *Proceedings of the 12th National Conference of Artificial Intelligence*, pages 1067–1072, 1994.

[13] C. B. Schwind. A logic based framework for action theories. In J. Ginzburg, Z. Khasidashvili, C. Vogel, J.-J. Lévy, and E. Vallduví, editors, *The Tbilisi Symposium on Logic, Language and Computation*, pages 275–291. 1997.

[14] M. Shanahan. *Solving the Frame Problem.* The MIT Press, 1997.

[15] M. Shanahan. The ramification problem in the event calculus. In C. S. Mellish, editor, *Proceedings of the International Joint Conference on Artificial Intelligence (IJCAI)*, pages 140–146. Morgan Kaufmann, 1999.

[16] Michael Thielscher. Ramification and causality. *Artificial Intelligence Journal*, 1997. (To appear. A preliminary version is available as Technical Report TR-96-003, ICSI, Berkeley, CA).