Pertinence for Causal Representation of Action Domains

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Outline

☞ **Introduction**

❑ Pertinence Logic L^2

❑ Action domains and transition systems

❑ $\mathcal{P}$-language: operational semantics

❑ Logical formalization

❑ Causal cycles

❑ Related work

❑ System PAL

❑ Conclusions

Scope

- Artificial Intelligence

- Reasoning about Actions and Change
  Programs with commonsense [McCarthy59]

- Logical representation of dynamic domains

- Key topics: inertia and causality.

Causality

A motivating example:

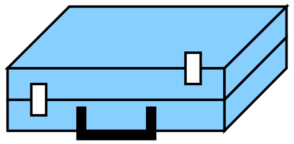
$$f = m \cdot a$$

$$\begin{aligned} a &= f/m \quad \text{(prediction)} \\ m &= f/a \quad \text{(explanation)} \\ f &= m \cdot a \quad \text{(planning)} \end{aligned}$$

m is not an effect of f and a.

Causality

Lin's suitcase.



If both locks are lifted up, the suitcase gets open

$$up(1) \wedge up(2) \supset open \tag{1}$$

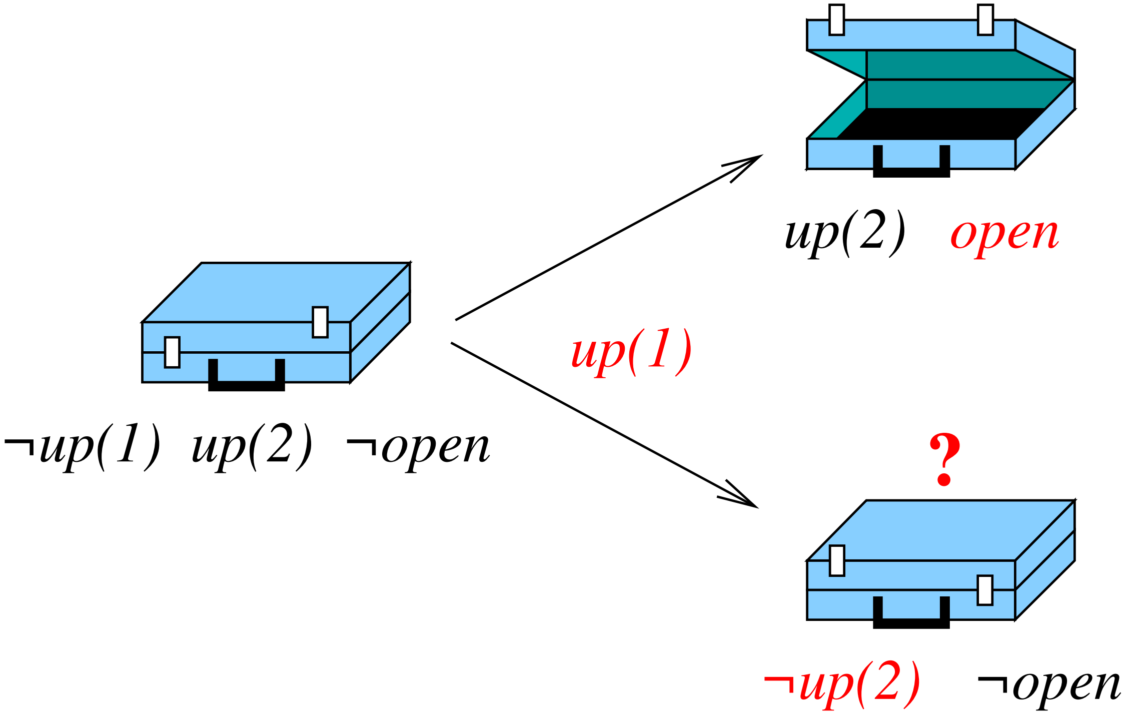
(prediction) . . . but it can also be "used" as:

$$up(1) \wedge \neg open \supset \neg up(2)$$

(explanation)

Problems when considering change

Example: change to a new state with $up(1)$ true (we lift lock 1), satisfying $up(1) \wedge up(2) \supset open$ but minimizing changes (inertia) ...



Solution: replace material implication by a causal conditional.

Causal approaches: two orientations

1. *Inferential* causality: causal conditionals $\approx$ inference rules.

2. *Change-based* causality:
   material implication $+$ a special predicate

   - "persisted" vs. "caused"
   - "caused" is auxiliary: only for ruling out undesired models

Our proposal: using both but providing ...

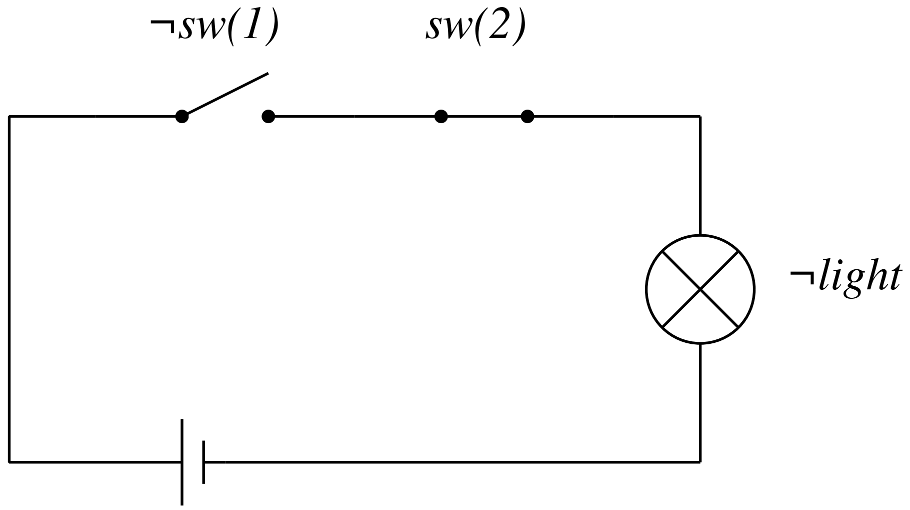
A meaning for “caused”

- Intuition: a fact is *caused* when it is obtained as a direct or indirect result of the performed actions.

  . . . otherwise, it simply persists.

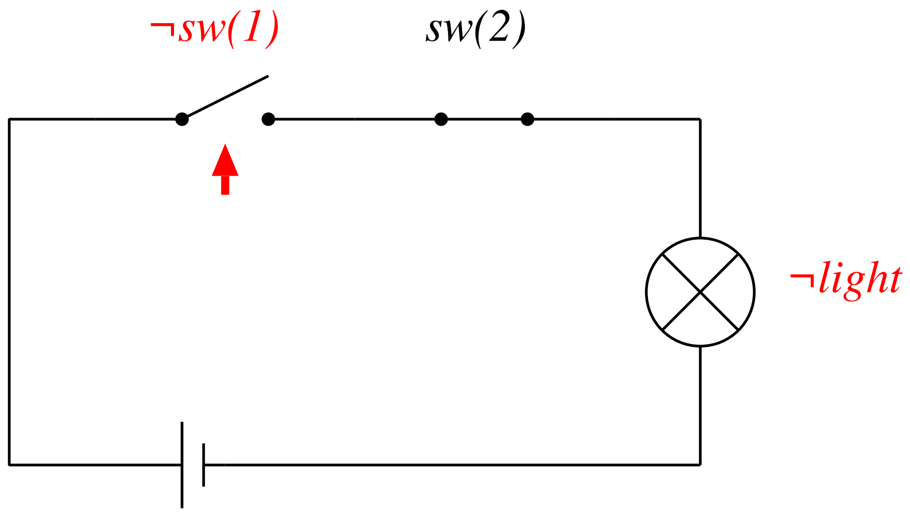
- Caused means “relevantly affected by the performed actions” : we will call it pertinent for short.

Example



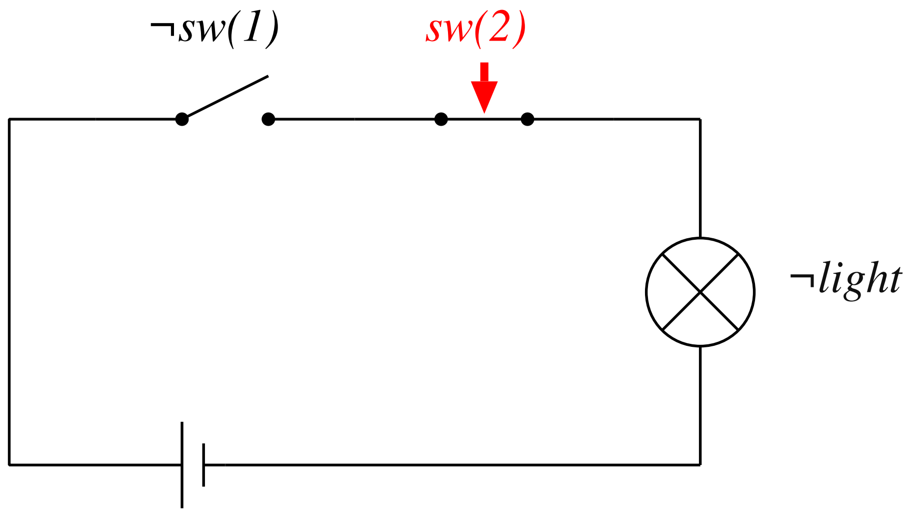
How could state $\neg sw(1), sw(2), \neg light$ have been obtained?

Example



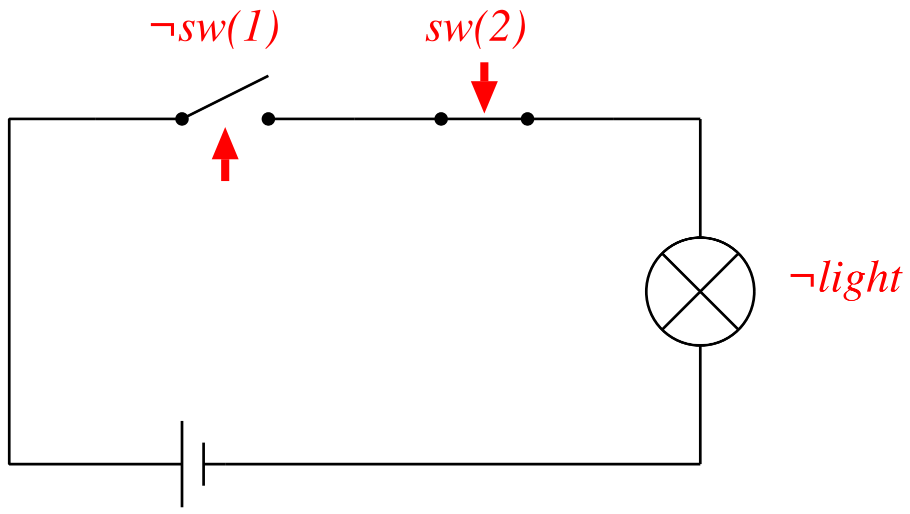
Opening $sw(1)$ while $sw(2)$ was closed $\Rightarrow$ $\neg light$ caused

Example



closing $sw(2)$ while $sw(1)$ was open $\Rightarrow$ $\neg light$ persisted

Example



Both opening $sw(1)$ and closing $sw(2)$ $\Rightarrow$ $\neg light$ caused

Intuitive postulates

P1) A fact is caused (or *pertinent*) whenever it is a direct or indirect effect of the performed actions.

P2) Any fact is obtained either by causal chaining or by inertia, but not both.

P3) A change of fluent value can only occur due to causation (*pertinence*).

P4) "Caused" does not necessarily mean a change of value.

P5) Conditions of causal rules are applicable iff they hold and are pertinent.

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- ☞ **Pertinence Logic L^2**
- ❑ Action domains and transition systems
- ❑ $\mathcal{P}$-language: operational semantics
- ❑ Logical formalization
- ❑ Causal cycles
- ❑ Related work
- ❑ System PAL
- ❑ Conclusions

Pertinence Logic L^2 [Otero97]

- Static and propositional. Finite signature $= \Sigma$.

- Syntax: propositional logic plus $\phi \Leftarrow \psi$

- Semantics: double valuation (truth and pertinence).
  Interesting: formulas can be pertinent.

- Interpretation, M is a pair $M^t M^p$ of functions s.t.
  for any $a \in \Sigma$, $M^t(a) \in \{\mathtt{t}, \mathtt{f}\}$ and $M^p(a) \in \{\mathtt{p}, \mathtt{n}\}$.

  For short, four values: $M(a) \in \{\mathtt{tp}, \mathtt{tn}, \mathtt{fp}, \mathtt{fn}\}$.

L^2 semantics

- Valuation of a formula α without $\Leftarrow$
  - $M^t(\alpha)$ is the standard truth-valuation.
  - $M^p(\alpha) = \mathtt{p}$ iff $a \in M^p$ for some atom a in ϕ.

- Example: $\Sigma = \{a, b, c, d, e\}$ $M_0^t = \{a, b, e\}$ $M_0^p = \{b, c, e\}$.

$$\begin{array}{cccccc} \neg & (& a & \vee & c &) \\ \mathtt{f} & & \mathtt{t} & \mathtt{t} & \mathtt{f} & \end{array} \qquad \mathtt{fp}$$

$$\begin{array}{cccc} a & \wedge & \neg & d \\ \mathtt{t} & \mathtt{t} & \mathtt{t} & \mathtt{f} \end{array} \qquad \mathtt{tn}$$

L^2 semantics

- Valuation of $\phi \Leftarrow \psi$
  - $M^t(\phi \Leftarrow \psi) = \mathtt{t}$ iff $M(\psi) \neq \mathtt{tp}$ or $M(\phi) = \mathtt{tp}$,
  - $M^p(\phi \Leftarrow \psi) = \mathtt{p}$ iff $M(\psi) = \mathtt{tp}$.

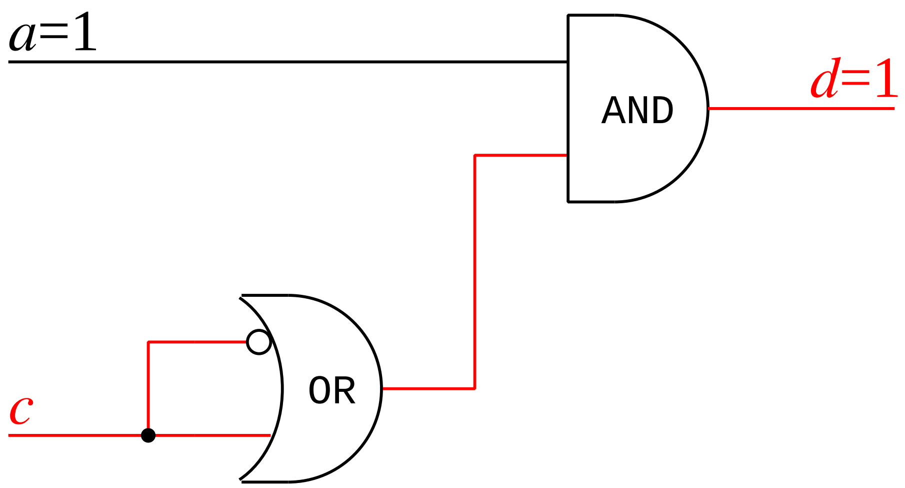
- Example: $M_0^t = \{a, b, e\}$ $M_0^p = \{b, c, e\}$.

$$\begin{array}{ccc} & \mathtt{t} & \\ e & \Leftarrow & b \\ \mathtt{tp} & & \mathtt{tp} \end{array} \qquad \begin{array}{ccc} & \mathtt{t} & \\ d & \Leftarrow & a \\ \mathtt{tn} & & \mathtt{tn} \end{array} \qquad \begin{array}{ccc} & \mathtt{f} & \\ d & \Leftarrow & e \\ \mathtt{tn} & & \mathtt{tp} \end{array}$$

- Model M of a theory T: for all $\phi \in T$, $M^t(\phi) = \mathtt{t}$.
- Truth-equivalence: $M^t(\phi) = M^t(\psi)$.

Relevance of tautologies

$$\begin{array}{ccc} & \mathtt{t} & \\ d & \Leftarrow & a \\ \mathtt{tn} & & \mathtt{tn} \end{array} \qquad\qquad \begin{array}{ccc} & \mathtt{f} & \\ d & \Leftarrow & a \wedge (c \vee \neg c) \\ \mathtt{tn} & & \mathtt{tp} \end{array}$$



- changing c, the persistence of d not guaranteed!

Other operators

$!\phi$	ϕ pertinent
$\mathbb{C}\,\phi \stackrel{\text{def}}{=} \phi \wedge !\phi$	ϕ has been caused
	$\phi \Leftarrow \psi$ is truth-equivalent to $\mathbb{C}\,\psi \supset \mathbb{C}\,\phi$
$\mathbb{P}\,\phi \stackrel{\text{def}}{=} \phi \wedge \neg !\phi$	ϕ has persisted
$\phi \vert \psi$	persistence-cut disjunction
$\phi \& \psi$	persistence-cut conjunction

- a set of transformation rules allows simplifying any L^2 theory into a classical one with atoms like:

$$\begin{array}{rcl} !a & \longrightarrow & pert(a) \\ a & \longrightarrow & holds(a) \end{array}$$

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Action domains

- fluents: $\mathcal{F} = \{up(1), up(2), open\}$.
- actions: $\mathcal{A} = \{toggle(1), toggle(2)\}$.
- symbol: any $p \in \mathcal{F} \cup \mathcal{A}$.
  $range(p) =$ possible values (usually boolean $\{\mathtt{t}, \mathtt{f}\}$).

Action domains

- Value fact: $holds(p, v)$. Boolean abbreviations

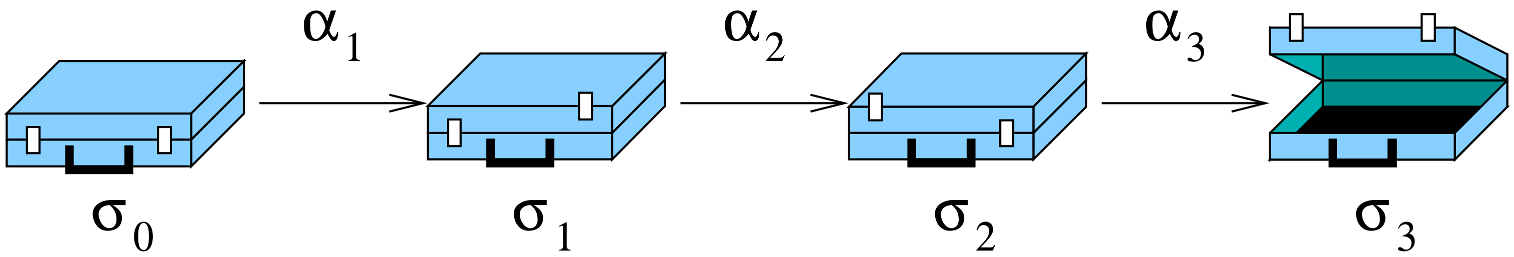
$$\begin{aligned} open &\stackrel{\text{def}}{=} holds(open, \mathtt{t}), \\ \overline{open} &\stackrel{\text{def}}{=} holds(open, \mathtt{f}) \end{aligned}$$

- State $\sigma : \mathcal{F} \longrightarrow \mathcal{V}$.
  Example: $\sigma_0 = \{\overline{up(1)}, up(2), \overline{open}\}$.

- Compound action $\alpha : \mathcal{A} \rightharpoonup \mathcal{V}$.
  Examples: $\alpha_1 = \{toggle(1)\}, \alpha_2 = \emptyset, \alpha_3 = \{toggle(1), toggle(2)\}$.

Narrative: $\sigma_0, \quad \alpha_1, \sigma_1, \quad \ldots, \quad \alpha_{n-1}, \sigma_{n-1}, \quad \alpha_n, \sigma_n$

$\{toggle(2)\}$ $\quad$ $\{toggle(1), toggle(2)\}$ $\quad$ $\{toggle(2)\}$



$\{\overline{up(1)}, \overline{up(2)}, \overline{open}\}$ $\quad$ $\{\overline{up(1)}, up(2), \overline{open}\}$ $\quad$ $\{up(1), \overline{up(2)}, \overline{open}\}$ $\quad$ $\{up(1), up(2), open\}$

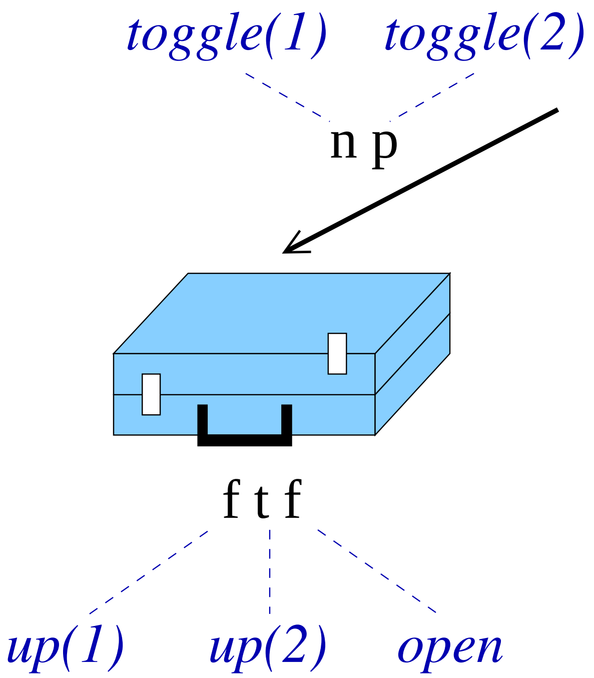
Atom: $holds(p, v, i)$, $i \in [0, n]$. Boolean abbreviations: p_i, $\overline{p_i}$.

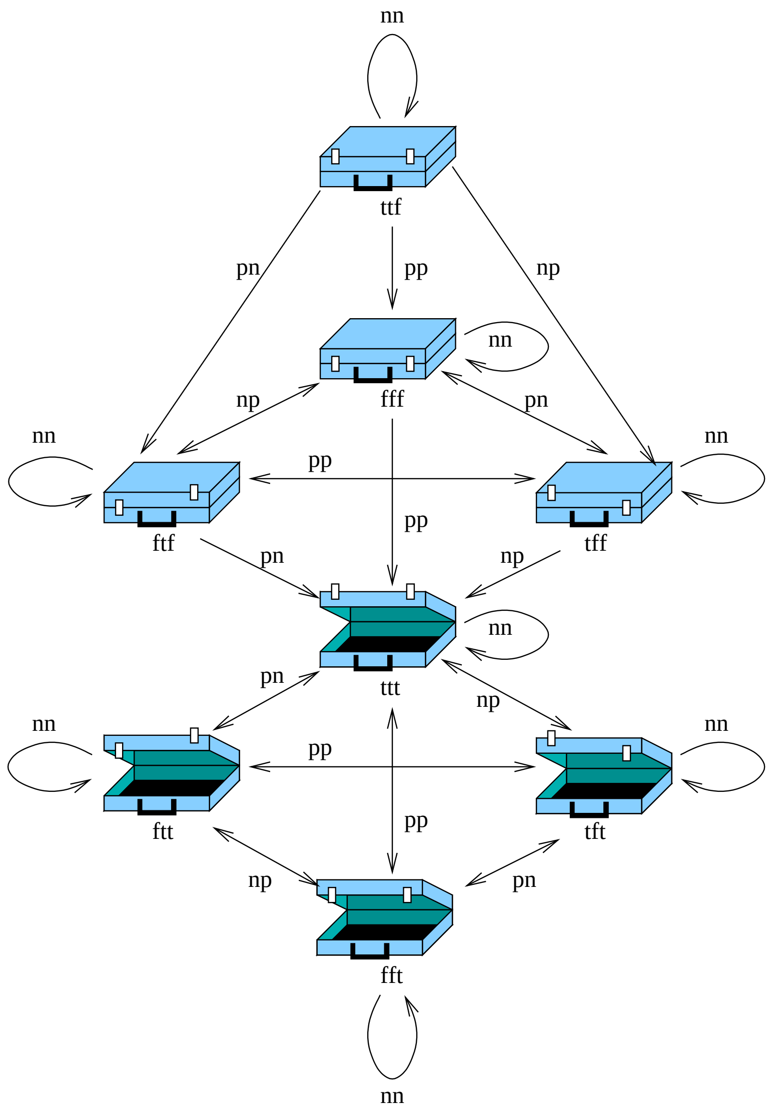
Transition systems

- σ_i only depends on σ_{i-1} and α_i.

- transition relation $\varrho \subseteq S \times A \times S$
  where $S =$ fluent states and $A =$ compound actions.

- We can use finite automata.


toggle(1)
toggle(2)
n p
f t f
up(1)
up(2)
open


nn
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nn
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np
nn
nn
pp
ftt
tft
pp
np
pn
fft
nn

Adding pertinence

- Pertinence facts $pert(p, v)$. $!p \stackrel{\text{def}}{=} pert(p, \mathtt{p})$ $\overline{!p} \stackrel{\text{def}}{=} pert(p, \mathtt{n})$

- Pertinence mapping $\pi : \mathcal{F} \cup \mathcal{A} \longrightarrow \{\mathtt{p}, \mathtt{n}\}$.

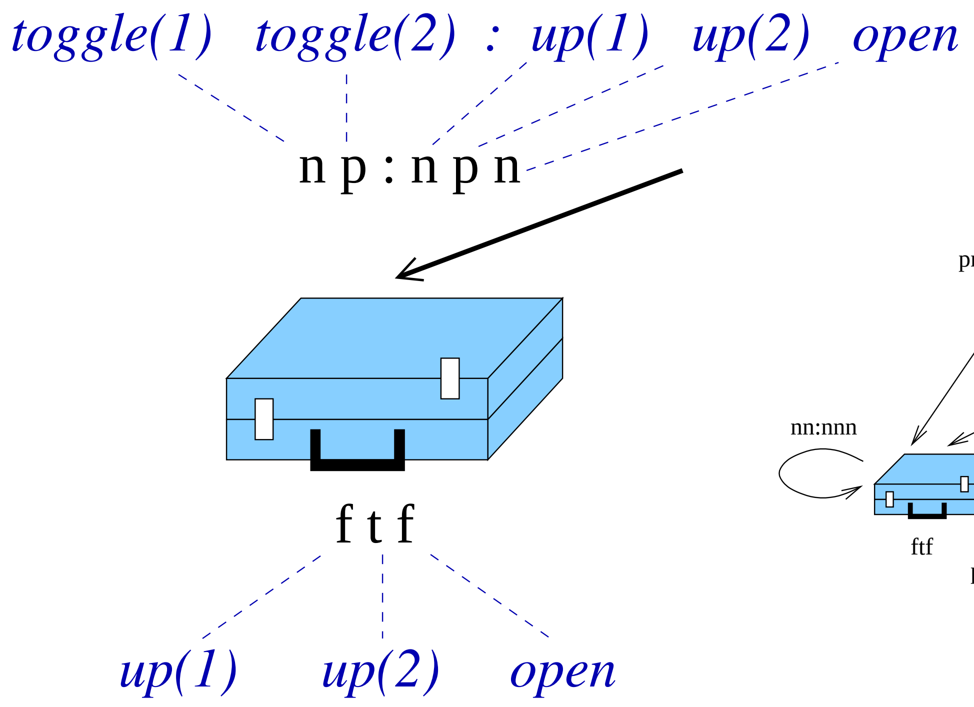
- Pertinence narrative: $\sigma_0, \quad \alpha_1, \sigma_1, \pi_1, \quad \ldots, \quad \alpha_n, \sigma_n, \pi_n$

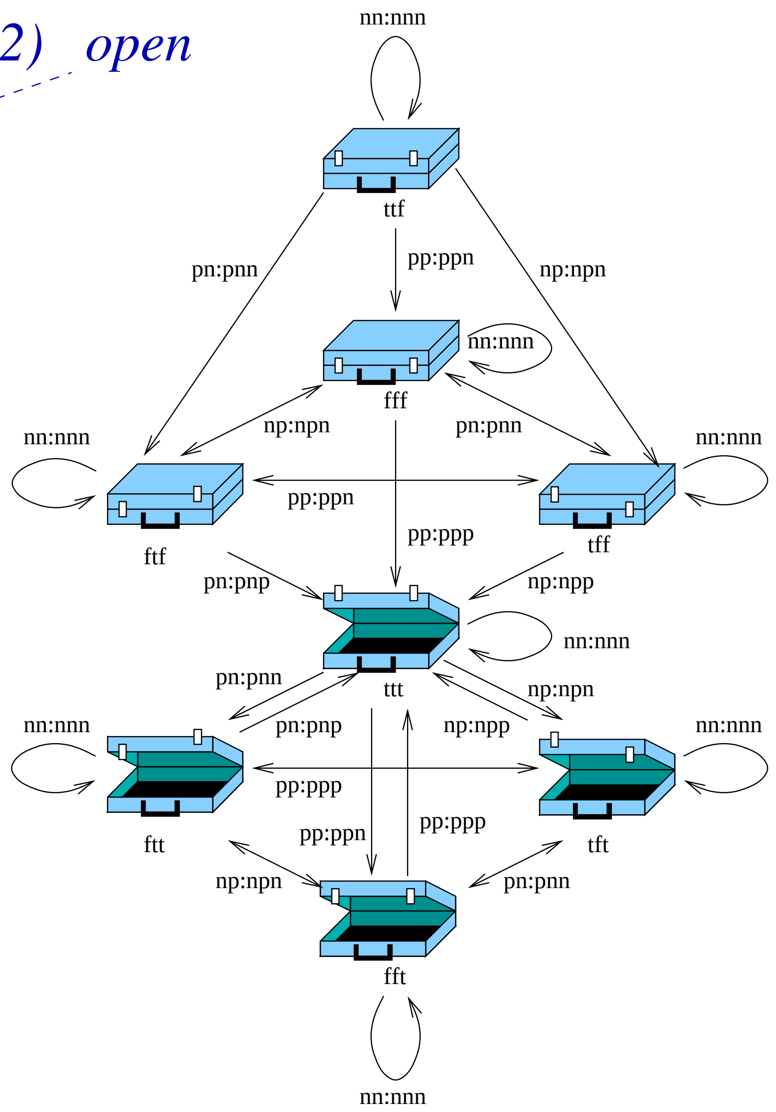
C1) an action is pertinent iff it is performed

C2) at 0, everything non-pertinent

C3) if $\pi_i(f) = \mathtt{n}$, then $\sigma_i(f) = \sigma_{i-1}(f)$.

- Transition systems. Pertinence as an output relation $\omega \subseteq S \times A \times P$ (where $P =$ pertinence mappings).


toggle(1) toggle(2) : up(1) up(2) open
n p : n p n
f t f
up(1) up(2) open


nn:nnn
ttf
pn:pnn
pp:ppn
np:npn
nn:nnn
fff
nn:nnn
np:npn
pn:pnn
nn:nnn
ftf
pp:ppn
pp:ppp
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pp:ppp
ftt
pp:ppn
pp:ppp
tft
np:npn
pn:pnn
fft
nn:nnn

Automata

- ✔ full description of system behavior
- ✔ prediction, explanation and planning is straightforward
- ✘ very sensitive to representation changes
  (no elaboration tolerance)
- ✘ no way to detect causal interactions

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$\mathcal{P}$-language

- Inspired by $\mathcal{A}$-language family [Gelfond&Lifschitz98]
- Rule shape: $\boxed{E \textbf{ if } C \textbf{ after } D}$

  E (effect): either $holds(f, v)$ or $\bot$,

  C (condition): $C_1 \wedge \cdots \wedge C_k$ (C_i facts of any kind),

  D (precondition): $D_1 \wedge \cdots \wedge D_m$ (D_i fluent value facts).

  We require no cycles among effects and conditions.

- Example (L=any lock):

$$
\begin{array}{cccccccc}
 & & & & up(L) & \textbf{if} & toggle(L) \textbf{ after } \overline{up(L)} & \\
 & & & & \overline{up(L)} & \textbf{if} & toggle(L) \textbf{ after } up(L) & \\
open & \textbf{if} & up(1) & \wedge & up(2) & & 0 & \leftarrow layer \\
2 & & 1 & & 1 & & &
\end{array}
$$

$\mathcal{P}$-language

- How to compute transitions? $\sigma_0, \alpha_1 \longrightarrow \sigma_1, \pi_1$
  increment a partial σ_1, π_1 applying rules layer by layer.

- A rule whose effect has layer j is applicable when:

  1. all the $D_i \in \sigma_0$, all $C_i \in \alpha_1$ (for actions)
  2. all the $C_i \in (\sigma_1 \cup \pi_1)$ up to $j - 1$
  3. for some symbol p in C: $!p \in \pi_1$ up to $j - 1$

Example

$\sigma_0 = \{\overline{up(1)}, up(2), \overline{open}\}$ $\alpha_1 = \{toggle(1)\}$

$$\begin{array}{lclclcll}
 & & & & up(L) & \textbf{if} & toggle(L) & \textbf{after}\ \overline{up(L)} \\
 & & & & \overline{up(L)} & \textbf{if} & toggle(L) & \textbf{after}\ up(L) \\
open & \textbf{if} & up(1) & \wedge & up(2) & & 0 & \leftarrow layer \\
2 & & 1 & & 1 & & &
\end{array}$$

layer j	Rule outcome	Persistence
0		$\emptyset$
1	$\{up(1)\}$	$\{up(2)\}$
2	$\{open\}$	$\emptyset$

procedure $transition(\sigma_0, \alpha_1,$**var** $\sigma_1,$**var** $\pi_1)$

begin

$Out := \emptyset;$

$Pers := \{holds(f, v) \in \sigma_0 \mid layer(f) = 0\};$

$\sigma_1 := Pers;$

$\pi_1 := \{!a \mid occurs(a, \alpha_1)\} \cup \{\overline{!a} \mid \text{not } occurs(a, \alpha_1)\} \cup$
$\{\overline{!(f)} \mid layer(f) = 0\};$

for j:=0 **to** $max(layer) - 1$ **do**

$Out := \{E \mid E$ effect of some applicable rule at layer $j\};$

if $inconsistent(Out)$ **or** $\perp \in Out$ **then**

halt("α_1 not executable");

$Pers := \{holds(f, v) \in \sigma_0 \mid layer(f) = j, \text{not } occurs(f, Out)\};$

$\sigma_1 := \sigma_1 \cup Out \cup Pers;$

$\pi_1 := \pi_1 \cup \{!f \mid occurs(f, Out)\} \cup \{\overline{!f} \mid occurs(f, Pers)\}$

end-for

end

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Pertinence Calculus: axioms AX

- Two predicates: $holds(F,V,I) \quad pert(F,V,I)$.
- Domain closure and unique names assumption for $\mathcal{A}, \mathcal{F}, \mathcal{V}, [0,n]$.
- Uniqueness and existence of value:

$$\forall P, I, V, V'.\ \Big(V \neq V' \supset \neg(holds(P,V,I) \wedge holds(P,V',I))\Big)$$

$$\forall F, I.\ \exists V.\ holds(F,V,I)$$

$$\forall P, I.\ \Big(pert(P,\mathtt{n},I) \equiv \neg pert(P,\mathtt{p},I)\Big)$$

- Conditions C1, C2, and C3 on narratives:

$$\forall A, I.\ \Big(pert(A,\mathtt{p},I) \equiv \exists V.\ holds(A,V,I)\Big)$$

$$\forall P.\ pert(P,\mathtt{n},0)$$

$$\forall F, V, I.\ \Big(\ I > 0\ \wedge pert(F,\mathtt{n},I) \wedge holds(F,V,I-1) \supset holds(F,V,I)\Big)$$

(UFR)

Pertinence calculus: rule translations $t(R)$

$\boxed{E \ \textbf{if}\ C \ \textbf{after}\ D}$ becomes $\boxed{\forall I.\big(I > 0 \land D_{I-1} \land \mathbb{C}\, C_I \supset \mathbb{C}\, E_I\big)}$

(L^2 operators $\mathbb{C}$, !, etc are "unfolded.")

$$\underbrace{up(1)}_{E}\ \textbf{if}\ \underbrace{toggle(1)}_{C}\ \textbf{after}\ \underbrace{\overline{up(1)}}_{D}$$

$$\Downarrow$$

$$\forall I.\big(I > 0 \land \underbrace{\overline{up(1)}_{I-1}}_{D_{I-1}} \land \underbrace{toggle(1)_I \land !toggle(1)_I}_{\mathbb{C}\, C_I} \supset \underbrace{up(1)_I \land !up(1)_I}_{\mathbb{C}\, E_I}\big)$$

$$\underbrace{open}_{E}\ \textbf{if}\ \underbrace{up(1) \land up(2)}_{C}$$

$$\Downarrow$$

$$\forall I.\big(I > 0 \land \underbrace{up(1)_I \land up(2)_I}_{C_I} \land \underbrace{(!up(1)_I \lor !up(2)_I)}_{!C_I} \supset \underbrace{open_I \land !open_I}_{\mathbb{C}\, E_I}\big)$$

Other constructions

For prediction problems,
we define complete action observations **do** (α, i).

$$\textbf{do}\ (\{toggle(2)\}, 4) \stackrel{\text{def}}{=} toggle(2)_4 \wedge \neg !toggle(1)_4$$

$$\textbf{do}\ (\emptyset, 5) \stackrel{\text{def}}{=} \neg !toggle(1)_5 \wedge \neg !toggle(2)_5$$

axioms+rules are not enough!

- Example: $\overline{up(1)}_0 \wedge up(2)_0 \wedge \overline{open}_0 \wedge$ **do** $(\{toggle(1)\}, 1)$

- Our logical theory yields 5 models (*transition* yields M_4)

	$up(2)_1$	$open_1$
M_1	`fp`	`fn`
M_2	`fp`	`fp`
M_3	`fp`	`tp`

	$up(2)_1$	$open_1$
M_4	`tn`	`tp`
M_5	`tp`	`tp`

- We need formulas for what does not change (frame axioms). Problems of elaboration tolerance: unrelated facts, concurrent actions, indirect effects.

- Result: obtaining frame axioms by "inspection" is unfeasible!

Solution: inertia default

- *"By default, fluents must remain unchanged."*
  ... but this means nonmonotonicity. Example:
  - When $dark\_room_0$, open the suitcase: $dark\_room_1 \in Cn(T)$.
  - Now, there is an internal lamp inside the suitcase:

$$dark\_room_1 \notin Cn(T \cup \{\overline{dark\_room} \textbf{ if } open\}) \tag{2}$$

- Techniques:
  - Circumscription
  - Default logic
  - Logic Programming: Stable Models, Completion, Well-founded semantics.

Circumscription [McCarthy80]

- Minimize the extent of a predicate ($pert$).

- Syntactically, a second order formula:

$$\mathrm{CIRC}[T; pert] \stackrel{\text{def}}{=} T(pert) \wedge \neg\exists P.\ (T(P) \wedge P < pert)$$

- Semantically, minimal models:

  take the less extent of $pert$ from models with fixed $holds$.

- We use the policy:
  $\mathrm{CIRC}[t(R) \cup Obs \cup \mathrm{AX}';\ pert] \quad \cup \quad (\mathrm{UFR})$

Circumscription

- Example:

$$\overline{up(1)}_0 \wedge up(2)_0 \wedge \overline{open}_0 \wedge \textbf{ do } (\{toggle(1)\}, 1)$$

- Models selection:

	$up(2)_1$	$open_1$	
M_6	fn	fn	minimal
M_7	fn	fp	
M_1	fp	fn	
M_2	fp	fp	
M_8	fn	tn	minimal
M_9	fn	tp	
M_{10}	fp	tn	
M_3	fp	tp	
M_4	tn	tp	minimal
M_5	tp	tp	

Circumscription

Theorem 7
Circumscription policy corresponds to the operational semantics, provided that rules do not contain $\overline{!p}$ in their conditions.

- Example: the alarm scenario.

$$\begin{array}{lll} active & \textbf{if} & connect \\ \overline{active} & \textbf{if} & disconnect \end{array} \qquad \begin{array}{rll} in & \textbf{if} & enter \\ ring & \textbf{if} & in \wedge active \wedge \overline{!active} \end{array}$$

- Executing action $\{enter\}$ we get a model with $M(active) = \mathtt{tp}$.

Default Logic [Reiter80]

- A default is a special inference rule of shape:

$$\frac{\alpha : \beta_1, \ldots, \beta_m}{\gamma}$$

- Reduct D^E (D set of defaults, E logically closed theory):

$$D^E = \left\{ \frac{\alpha}{\gamma} \;\middle|\; \frac{\alpha : \beta_1, \ldots, \beta_m}{\gamma} \in D, \text{ and there is no } \neg\beta_j \in E \right\}$$

- D^E only contains "usual" inference rules.
  $Cn(D^E)$ denotes its deductive closure.

- E is an extension of D iff $E = Cn(D^E)$.

Pertinence in Default Logic

Rule translation

$$\frac{D_{I-1} \wedge \mathbb{C}\, C_I}{\mathbb{C}\, E_I}$$

(UFR)

$$\frac{pert(F, \mathtt{n}, I) \wedge holds(F, V, I-1)}{holds(F, V, I)}$$

Pertinence minimization

$$\frac{: pert(P, \mathtt{n}, I)}{pert(P, \mathtt{n}, I)}$$

- Default Logic fully captures the operational behavior, but for practical purposes, we can use its strong connection to stable models.

Logic Programming (LP).

A pertinence calculus theory will become a logic program to be interpreted under:

1. Stable models

2. Clark's completion

3. Well-founded semantics

Stable models [Gelfond&Lifschitz88]

- Like default theories, but restricted to atoms:

  $\dfrac{a_1 \wedge a_2 : \neg b_1, \neg b_2}{c}$ becomes the rule $c \leftarrow a_1, a_2, \mathit{not}\ b_1, \mathit{not}\ b_2$

- Reduct P^M (P set of rules, M set of ground atoms):

  $$P^M = \{(c \leftarrow a_1, \ldots, a_n) \mid (c \leftarrow a_1, \ldots, a_n, \mathit{not}\ b_1, \ldots, \mathit{not}\ b_m) \in P, \text{ and all } b_j \notin M\}$$

- P^M has not default negation. It has a least model (closure of direct consequences) written $\Gamma_P(M)$.

- Stable models are fixpoints: $M = \Gamma_P(M)$.

Stable models

Some properties:

- We may get several stable models:

$$\begin{array}{rcl} p & \leftarrow & \textit{not}\ q \\ q & \leftarrow & \textit{not}\ p \end{array}$$

- ... or no stable model at all:

$$p \leftarrow \textit{not}\ p$$

- ... but only in presence of "negative" cycles.

Well-founded semantics (WFS) [van Gelder et al. 91]

- Γ_P^2 has a least and a greatest fixpoint.
  (both computable by iteration).

- Well-founded model (WFM):
  - True atoms: $lfp(\Gamma_P^2)$
  - False atoms: $\Sigma - gfp(\Gamma_P^2)$
  - the rest are left undefined (due to negative cycles)

- defined atoms in the WFM are fixed in all the stable models.

- For explicit negation $\overline{p}$, a variant is needed:
  WFSX [Alferes&Pereira92].

Clark's completion [Clark78]

- Consider the program P_1:

$$\begin{array}{lll} p & \leftarrow & q, r \\ p & \leftarrow & s \\ r & & \end{array} \qquad\qquad \begin{array}{lll} P_1 & \not\models & p \\ P_1 & \not\models & \neg p \end{array}$$

- Idea: rules are the only ways to obtain head atoms.
  $\mathrm{COMP}[P_1] = \{p \equiv (q \wedge r) \vee s, \quad r \equiv \top, \quad q \equiv \bot, \quad s \equiv \bot\}$

- Its models (called supported) are the fixpoints of

$$T_P(M) = \{c \mid (c \leftarrow Body) \in P \text{ and } M \models Body\}$$

- Positive cycles: $\{p \leftarrow p\}$ has 2 supported models $\{p\}$ and $\emptyset$.

Logic Programming

- A program is hierarchical iff it contains no cycles.

- Interesting property:

  For a hierarchical programs, stable models, Clark's completion and WFS yield the same unique model.

Pertinence in Logic Programming

- Axioms:

$$
\begin{array}{rcl}
\bot & \leftarrow & holds(P,V,I),\ holds(P,V',I) \\
\bot & \leftarrow & pert(P,\mathtt{p},I),\ pert(P,\mathtt{n},I) \\
pert(A,\mathtt{p},I) & \leftarrow & holds(A,V,I) \\
holds(F,V,I) & \leftarrow & pert(F,\mathtt{n},I),\ holds(F,V,I-1)
\end{array} \qquad \text{(UFR)}
$$

- Pertinence minimization: $pert(P,\mathtt{n},I) \leftarrow$ *not* $pert(P,\mathtt{p},I)$

- Example of $\mathcal{P}$-rule translation: $open$ **if** $up(1) \wedge up(2)$ becomes

$$
\begin{array}{rcl}
open_I & \leftarrow & up(1)_I,\ up(2)_I,\ !up(1)_I \\
!open_I & \leftarrow & up(1)_I,\ up(2)_I,\ !up(1)_I \\
open_I & \leftarrow & up(1)_I,\ up(2)_I,\ !up(2)_I \\
!open_I & \leftarrow & up(1)_I,\ up(2)_I,\ !up(2)_I
\end{array}
$$

Pertinence in Logic Programming

Theorem 8
Given a set of acyclic $\mathcal{P}$-rules R, the corresponding logic program is hierarchical.

Proof sketch: the resulting program $P(R)$ can be arranged into different levels (3 levels per layer).

Theorem 9
The logic programming encoding under (any of the 3 LP semantics) corresponds to the operational semantics.

Proof sketch: proceed by induction on situations and layers, using WFS iterative construction.

Then, why 3 semantics? $\Rightarrow$ they differ for cycles.

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- ✔ Introduction
- ✔ Pertinence Logic L^2
- ✔ Action domains and transition systems
- ✔ $\mathcal{P}$-language: operational semantics
- ✔ Logical formalization
- ☞ **Causal cycles**
- ❑ Related work
- ❑ System PAL
- ❑ Conclusions

Causal cycles

- Cyclic $\mathcal{P}$-rules $\Rightarrow$ layer-based algorithm not applicable.

- Depending on the non-monotonic technique, we may obtain: non-existence of model, non-determinism, undefined values.

- Most effects will depend on the studied transition (cycles behave dynamically).

Self-supportedness

- Problem: the ultimate justification for an effect should not be the effect itself.

- Example: having $\overline{b}_0 \wedge$ **do** $(\emptyset, 1)$ and rule: $\boxed{b \textbf{ if } b}$

  1. Circumscription, WFS, stable models: b persists false.

  2. Completion yields 2 models (non-determinism):
     (the one above + b becomes caused true!).

- b is "self-justified" without external intervention.

Self-supportedness

- Also in circumscription ...

- Example: $\boxed{b \textbf{ if } b \wedge a}$ with $\bar{b}_0 \wedge$ **do** $(\{a\}, 1)$.

  1. WFS, stable models: b persists false.

  2. Completion, Circumscription: additional model with b caused true.

- Important property: stable models guarantee absence of self-supportedness (and in fact, the same applies for WFS).

Non-existence of model

- Example: $\boxed{b \textbf{ if } b \wedge a}$ with $b_0 \wedge$ **do** $(\{a\}, 1)$.

  1. stable models: no stable model.

  2. WFS: atoms b_1, $!b_1$ and $\overline{!b_1}$ become undefined.

  3. Circumscription, completion: b caused true (self-supported).

- Using these kind of cycles for non-executability of actions is not a good idea:

  - their behavior depends on the previous situation,

  - we can use: $\perp$ **if** a **after** b

  Better to use this as hint of bad design.

WFS always “tries” determinism

$$\boxed{\begin{array}{lll} b & \textbf{if} & \overline{c} \wedge a \\ c & \textbf{if} & \overline{b} \wedge a \end{array}}$$

$$\overline{b}_0 \wedge \overline{c}_0 \wedge \ \textbf{do}\ (\{a\}, 1)$$

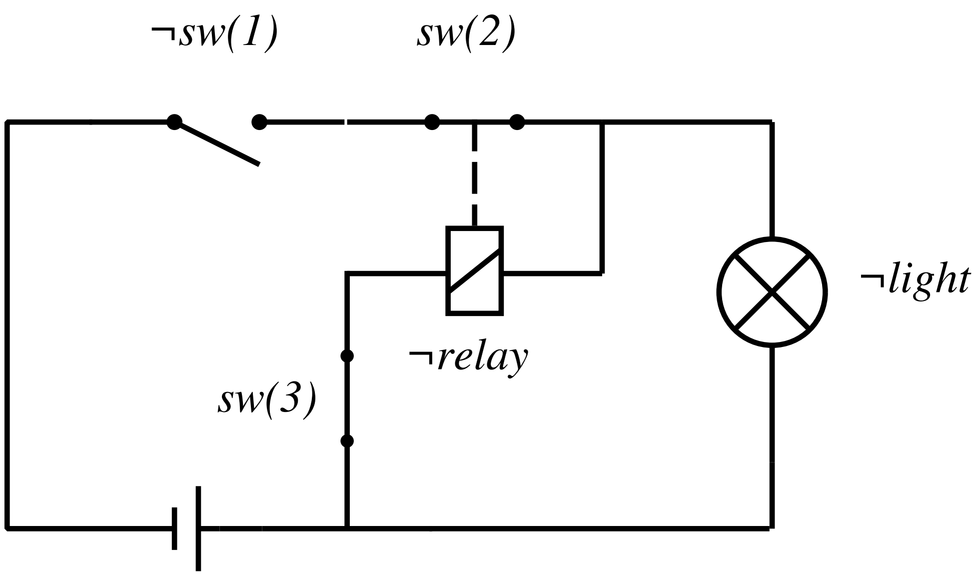
- Result:

  1. WFS: b and c undefined.
  2. the rest yield 2 solutions:
     b persists false, c caused true; and vice versa.

- For other initial states, all semantics agree.

- Again: unclear way for intended non-determinism.
  It seems to point out bad design.

Example of “realistic” cycle problem



Relay circuit [Shanahan99]

- Toggling switch 1 at the situation above:
    1. WFS: $sw(2), relay$ and $light$ undefined ($sw(1)$ caused true).
    2. the rest: no model.

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Related action approaches

1. Change as abnormality
2. Lin's $caused$ predicate
3. Occlusion/TAL
4. Event Calculus
5. Inductive Causation
6. Abnormality in Logic Programming
7. Thielscher's Causal Relationships
8. Causal Explanation
9. Schwind's comparative

2. Lin's *caused* predicate [Lin95]

- Truth value reified:

$$\begin{aligned} caused(F, \mathtt{t}, S) &\supset holds(F, S) \\ caused(F, \mathtt{f}, S) &\supset \neg holds(F, S) \end{aligned}$$

- $caused$ versus persistent:

$$\neg caused(F, \mathtt{t}, S) \wedge \neg caused(F, \mathtt{f}, S) \supset [holds(F, do(A, S)) \equiv holds(F, S)] \quad \text{(UFR)}$$

- Similar circumscription policy (leave (UFR) outside).
- no P5: conditions do not refer to $caused$

$$holds(up(1), S) \wedge holds(up(2), S) \supset caused(open, \mathtt{t}, S)$$

- no P1: waiting, when the suitcase is already open.
- unlike $pert$, $caused$ is not defined for formulas.

3. Occlusion [Sandewall94]/TAL [Gustafsson&Doherty96]

- In [Sandewall94] (page 235), we find P4:
  "...occlusion does not *imply* change; it merely allows it."

$$\neg occlude(F, I) \supset holds(F, I-1) \equiv holds(F, I) \qquad \text{(UFR)}$$

- Same circumscription (minimize $occlude$ with (UFR) outside).

- In low-level notation, the rule for $open$ becomes:

$$\begin{aligned} \overbrace{holds(up(1), I) \wedge holds(up(2), I)}^{C_I} \quad &\supset \quad holds(open, I) \\ C_I \wedge \neg C_{I-1} \quad &\supset \quad occlude(open, I) \end{aligned}$$

- P5 not satisfied, but change of truth value in the condition.
  Occlusion of ϕ: all fluents in ϕ occluded ($\neq!\phi$).

5. Inductive Causation [Denecker et al.98].

- P1 is satisfied ([Denecker et al.98] section 5.3, page 24):

  "...an effect is never triggered by the absence of other effects alone, e.g. the suitcase will not be open just because nothing happens."

- Alternative underlying non-monotonic techniques (cycles).

- $\boxed{\textbf{initiating } up(1) \wedge up(2) \textbf{ causes } open}$ becomes:

$$
\begin{aligned}
caus(open) &\leftarrow init(up(1)) \wedge holds(up(2)) \wedge \neg init(\neg up(2)) \\
caus(open) &\leftarrow init(up(2)) \wedge holds(up(1)) \wedge \neg init(\neg up(1)) \\
caus(open) &\leftarrow init(up(1)) \wedge init(up(2))
\end{aligned}
$$

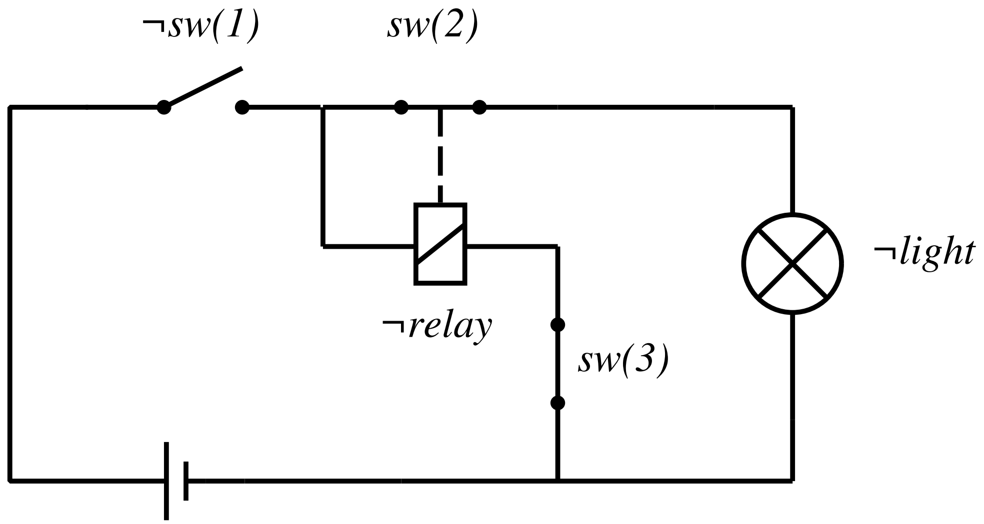
- Initiation of a formula $\Rightarrow$ close to pertinence!

- However, no P5: *init* means an actual change of value.

6. Causal Relationships [Thielscher97].

$$\boxed{\begin{array}{l} up(1) \textbf{ causes } open \textbf{ if } up(2) \\ up(2) \textbf{ causes } open \textbf{ if } up(1) \end{array}}$$

- Algorithmic: trigger rules with intermediate states.
- During the process, fluent may change its value.
- It can be non-deterministic even with acyclic rules.



7. Causal Explanation [McCain&Turner97].

- Strongly related to completion/supported models.

- $\phi \leftarrow \psi \quad \approx \quad \psi$ true implies ϕ "caused."

- Strictly inferential (no change predicate).

$$\begin{array}{rcl} holds(open, I) & \leftarrow & holds(up(1), I) \wedge holds(up(2), I) \\ holds(F, I+1) & \leftarrow & holds(F, I+1) \wedge holds(F, I) \\ \neg holds(F, I+1) & \leftarrow & \neg holds(F, I+1) \wedge \neg holds(F, I) \end{array}$$

- Important: "*caused*" has a different meaning here.
  Everything must get finally "caused," including inertia!

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PAL

- Extended language: exploit functional notation

```
up(L):=not prev(up(L)) if toggle(L);
```

- Main modules (programmed in `C`):

1. Rule/query grounding:
   - ground $\mathcal{P}$-rules
   - logic programs in SMODELS internal format (`C++`)
2. Action generator (for temporal queries)
3. WFSX algorithm

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Conclusions

- Methodology: begin with automata, $\mathcal{P}$-language and operational semantics.
- Logical formalization at 2 levels:
  1. Theoretical foundation: pertinence, L^2.
  2. Several alternatives for nonmonotonic inference (circumscription, stable models, completion, WFS).
- Comparisons (cycles, other action aproaches).
- PAL interpreter.

Future work

1) Usual topics in Reasoning about Actions:
(Explicit) nondeterminism, qualification problem, natural actions, actions with durations, sensing and robot knowledge.

2) Other interesting topics:

- Strong equivalence between causal representations. [Lifschitz,Pearce,Valverde00]
- Proving temporal properties.
- Machine Learning [Lorenzo01].

3) PAL: improvements + extensions.

Resumen