Explaining Answer Sets in Argumentative Terms

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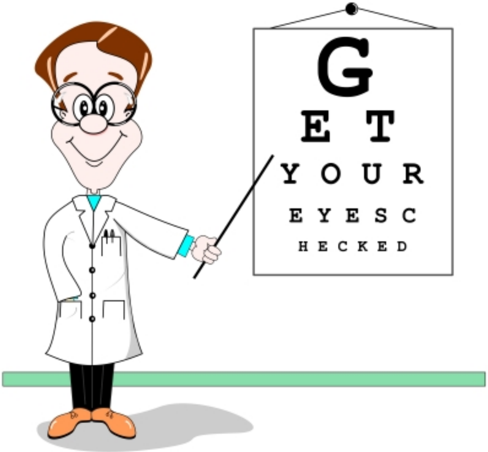
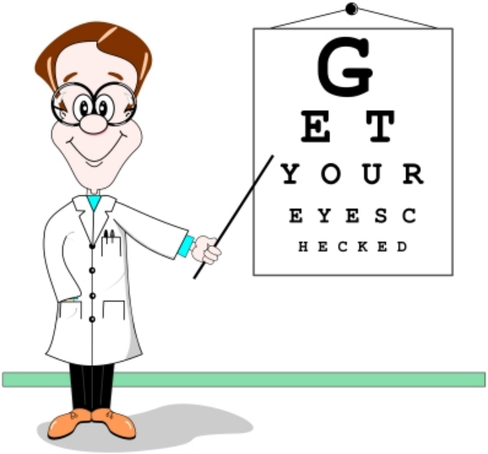
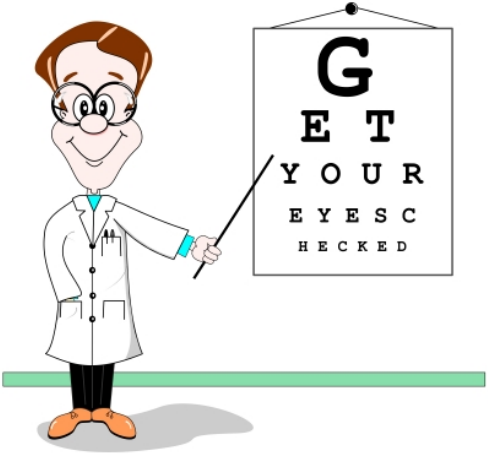

G
E T
Y O U R
E Y E S C
H E C K E D

patient is shortsighted



patient is shortsighted

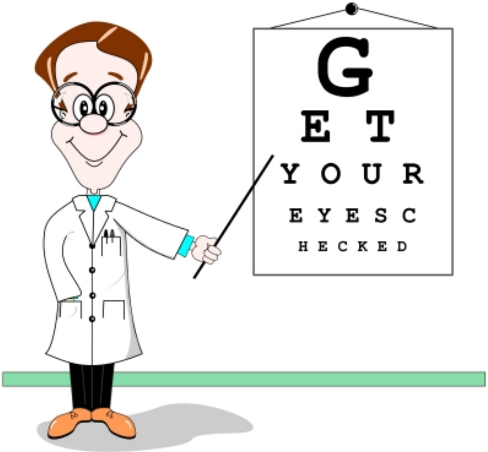
glasses?
laser surgery?
contact lenses?
intraocular lenses?



patient is shortsighted

glasses?
laser surgery?
contact lenses?
intraocular lenses?

further patient info:

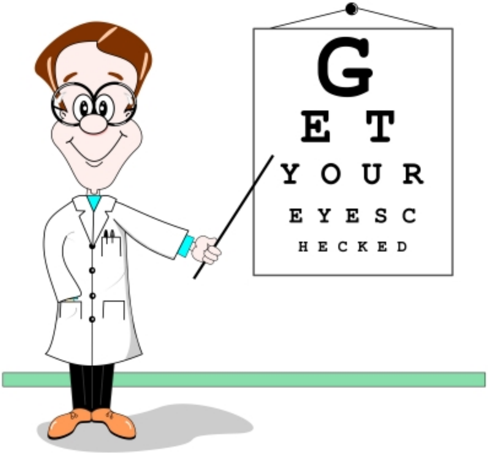


patient is shortsighted

glasses?
laser surgery?
contact lenses?
intraocular lenses?

further patient info:

- likes sports
- afraid to touch eyes
- student



laser surgery!

laser surgery!

Answer Set Programming
(ASP)

A logic program

$$
\begin{array}{lcl}
\mathit{tightOnMoney} & \leftarrow & \mathit{student}, \mathit{not}\ \mathit{richParents} \\
\mathit{caresAboutPracticality} & \leftarrow & \mathit{likesSports}
\end{array}
$$

A logic program

$$
\begin{array}{lcl}
\mathit{tightOnMoney} & \leftarrow & \mathit{student}, \text{ not } \mathit{richParents} \\
\mathit{caresAboutPracticality} & \leftarrow & \mathit{likesSports} \\
\\
\mathit{correctiveLens} & \leftarrow & \mathit{shortSighted}, \text{ not } \mathit{laserSurgery} \\
\mathit{laserSurgery} & \leftarrow & \mathit{shortSighted}, \text{ not } \mathit{tightOnMoney}, \\
 & & \text{not } \mathit{correctiveLens}
\end{array}
$$

A logic program

$$
\begin{array}{lcl}
\mathit{tightOnMoney} & \leftarrow & \mathit{student}, \text{ not } \mathit{richParents} \\
\mathit{caresAboutPracticality} & \leftarrow & \mathit{likesSports} \\
\\
\mathit{correctiveLens} & \leftarrow & \mathit{shortSighted}, \text{ not } \mathit{laserSurgery} \\
\mathit{laserSurgery} & \leftarrow & \mathit{shortSighted}, \text{ not } \mathit{tightOnMoney}, \\
 & & \text{not } \mathit{correctiveLens} \\
\\
\mathit{glasses} & \leftarrow & \mathit{correctiveLens}, \\
 & & \text{not } \mathit{caresAboutPracticality}, \\
 & & \text{not } \mathit{contactLens} \\
\mathit{contactLens} & \leftarrow & \mathit{correctiveLens}, \\
 & & \text{not } \mathit{afraidToTouchEyes}, \\
 & & \text{not } \mathit{longSighted}, \text{ not } \mathit{glasses} \\
\mathit{intraocularLens} & \leftarrow & \mathit{correctiveLens}, \text{ not } \mathit{glasses}, \\
 & & \text{not } \mathit{contactLens}
\end{array}
$$

A logic program

$$
\begin{array}{lll}
\textit{tightOnMoney} & \leftarrow & \textit{student}, \textit{not richParents} \\
\textit{caresAboutPracticality} & \leftarrow & \textit{likesSports} \\
\\
\textit{correctiveLens} & \leftarrow & \textit{shortSighted}, \textit{not laserSurgery} \\
\textit{laserSurgery} & \leftarrow & \textit{shortSighted}, \textit{not tightOnMoney}, \\
 & & \textit{not correctiveLens} \\
\\
\textit{glasses} & \leftarrow & \textit{correctiveLens}, \\
 & & \textit{not caresAboutPracticality}, \\
 & & \textit{not contactLens} \\
\textit{contactLens} & \leftarrow & \textit{correctiveLens}, \\
 & & \textit{not afraidToTouchEyes}, \\
 & & \textit{not longSighted}, \textit{not glasses} \\
\textit{intraocularLens} & \leftarrow & \textit{correctiveLens}, \textit{not glasses}, \\
 & & \textit{not contactLens} \\
\\
\textit{shortSighted} & \leftarrow & \\
\textit{afraidToTouchEyes} & \leftarrow & \\
\textit{student} & \leftarrow & \\
\textit{likesSports} & \leftarrow &
\end{array}
$$

A logic program

$$
\begin{array}{lcl}
\mathit{tightOnMoney} & \leftarrow & \mathit{student}, \text{ not } \mathit{richParents} \\
\mathit{caresAboutPracticality} & \leftarrow & \mathit{likesSports} \\
\\
\mathit{correctiveLens} & \leftarrow & \mathit{shortSighted}, \text{ not } \mathit{laserSurgery} \\
\mathit{laserSurgery} & \leftarrow & \mathit{shortSighted}, \text{ not } \mathit{tightOnMoney}, \\
 & & \text{not } \mathit{correctiveLens} \\
\\
\mathit{glasses} & \leftarrow & \mathit{correctiveLens}, \\
 & & \text{not } \mathit{caresAboutPracticality}, \\
 & & \text{not } \mathit{contactLens} \\
\mathit{contactLens} & \leftarrow & \mathit{correctiveLens}, \\
 & & \text{not } \mathit{afraidToTouchEyes}, \\
 & & \text{not } \mathit{longSighted}, \text{ not } \mathit{glasses} \\
\mathit{intraocularLens} & \leftarrow & \mathit{correctiveLens}, \text{ not } \mathit{glasses}, \\
 & & \text{not } \mathit{contactLens} \\
\\
\mathit{shortSighted} & \leftarrow & \\
\mathit{afraidToTouchEyes} & \leftarrow & \\
\mathit{student} & \leftarrow & \\
\mathit{likesSports} & \leftarrow &
\end{array}
$$

Answer Set:

$\{\mathit{shortSighted}, \mathit{afraidToTouchEyes}, \mathit{student}, \mathit{likesSports}, \mathit{tightOnMoney}, \mathit{correctiveLens}, \mathit{caresAboutPracticality}, \mathit{intraocularLens}\}$

A logic program

$$
\begin{aligned}
tightOnMoney &\leftarrow student, \text{not } richParents\\
caresAboutPracticality &\leftarrow likesSports\\[1ex]
correctiveLens &\leftarrow shortSighted, \text{not } laserSurgery\\
laserSurgery &\leftarrow shortSighted, \text{not } tightOnMoney,\\
&\qquad \text{not } correctiveLens\\[1ex]
glasses &\leftarrow correctiveLens,\\
&\qquad \text{not } caresAboutPracticality,\\
&\qquad \text{not } contactLens\\
contactLens &\leftarrow correctiveLens,\\
&\qquad \text{not } afraidToTouchEyes,\\
&\qquad \text{not } longSighted, \text{not } glasses\\
intraocularLens &\leftarrow correctiveLens, \text{not } glasses,\\
&\qquad \text{not } contactLens\\[1ex]
shortSighted &\leftarrow\\
afraidToTouchEyes &\leftarrow\\
student &\leftarrow\\
likesSports &\leftarrow
\end{aligned}
$$

Answer Set:

$\{shortSighted, afraidToTouchEyes, student, likesSports,$
$tightOnMoney, correctiveLens, caresAboutPracticality, intraocularLens\}$

laser surgery!

Answer Set Programming
(ASP)

intraocular lenses!

Why is "intraocular lenses" a solution?

laser surgery!

Answer Set Programming
(ASP)

intraocular lenses!

Why is “intraocular lenses” a solution?

$\Rightarrow$ **Explain why something is (not) in an answer set**

ASP and Argumentation

Example (Answer Set Programming)

$$
\begin{array}{lcl}
a & \leftarrow & \mathit{not}\ \neg a \\
a & \leftarrow & \neg a, \mathit{not}\ c, \mathit{not}\ e \\
\neg a & \leftarrow & \mathit{not}\ c, \mathit{not}\ d \\
c & \leftarrow & \mathit{not}\ e \\
d & \leftarrow & \mathit{not}\ \neg a \\
e & \leftarrow &
\end{array}
$$

Example (Answer Set Programming)

$$
\begin{array}{lcl}
a & \leftarrow & not\ \neg a \\
a & \leftarrow & \neg a, not\ c, not\ e \\
\neg a & \leftarrow & not\ c, not\ d \\
c & \leftarrow & not\ e \\
d & \leftarrow & not\ \neg a \\
e & \leftarrow &
\end{array}
$$

$e \in S$

ASP and Argumentation

Example (Answer Set Programming)

$$
\begin{array}{ccl}
a & \leftarrow & \mathit{not}\ \neg a \\
a & \leftarrow & \neg a, \mathit{not}\ c, \mathit{not}\ e \\
\neg a & \leftarrow & \mathit{not}\ c, \mathit{not}\ d \\
c & \leftarrow & \mathit{not}\ e \\
d & \leftarrow & \mathit{not}\ \neg a \\
e & \leftarrow &
\end{array}
$$

$e \in S$

ASP and Argumentation

Example (Answer Set Programming)

$$
\begin{array}{rcl}
a & \leftarrow & not\ \neg a \\
\neg a & \leftarrow & not\ c, not\ d \\
d & \leftarrow & not\ \neg a \\
e & \leftarrow &
\end{array}
$$

$e \in S$

ASP and Argumentation

Example (Answer Set Programming)

$$\begin{array}{rcl} a & \leftarrow & not\ \neg a \\ \neg a & \leftarrow & not\ c, not\ d \\ d & \leftarrow & not\ \neg a \\ e & \leftarrow & \end{array}$$

$e \in S$

$d \in S?$

ASP and Argumentation

Example (Answer Set Programming)

$$\begin{array}{lcl} a & \leftarrow & \mathit{not}\ \neg a \\ \neg a & \leftarrow & \mathit{not}\ c, \mathit{not}\ d \\ d & \leftarrow & \mathit{not}\ \neg a \\ e & \leftarrow & \end{array}$$

$e \in S$

$d \in S$?

ASP and Argumentation

Example (Answer Set Programming)

$$
\begin{array}{lll}
a & \leftarrow & not\ \neg a \\
 & & \\
d & \leftarrow & not\ \neg a \\
e & \leftarrow &
\end{array}
$$

$$e \in S$$

$$d \in S?$$

Example (Answer Set Programming)

$$\begin{array}{rcl} a & \leftarrow & not\ \neg a \\ & & \\ d & \leftarrow & not\ \neg a \\ e & \leftarrow & \end{array}$$

$$e \in S$$
$$d \in S?$$
$$S = \{e, d, a\}$$

ASP and Argumentation

Example (Answer Set Programming)

$$\begin{array}{lll} a & \leftarrow & not\ \neg a \\ \\ d & \leftarrow & not\ \neg a \\ e & \leftarrow & \end{array}$$

$$e \in S$$
$$d \in S?$$
$$S = \{e, d, a\}$$

Interaction between **classical literals** and **NAF literals**!

ASP and Argumentation

Example (Answer Set Programming)

$$a \leftarrow not\ \neg a$$

$$d \leftarrow not\ \neg a$$

$$e \leftarrow$$

$$e \in S$$

$$d \in S?$$

$$S = \{e, d, a\}$$

Interaction from **classical literals** to **NAF literals**!

ASP and Argumentation

Example (Answer Set Programming)

$$\begin{array}{lll} a & \leftarrow & not\ \neg a \\ & & \\ d & \leftarrow & not\ \neg a \\ e & \leftarrow & \end{array}$$

$$e \in S$$
$$d \in S?$$
$$S = \{e, d, a\}$$

Interaction from **classical literals** to **NAF literals**!

Example (Assumption-Based Argumentation (ABA))

- **rules** = Answer Set Program

ASP and Argumentation

Example (Answer Set Programming)

$$\begin{array}{lcl} a & \leftarrow & not\ \neg a \\ & & \\ d & \leftarrow & not\ \neg a \\ e & \leftarrow & \end{array}$$

$$e \in S$$
$$d \in S?$$
$$S = \{e, d, a\}$$

Interaction from **classical literals** to **NAF literals**!

Example (Assumption-Based Argumentation (ABA))

- **rules** = Answer Set Program
- **assumptions** = NAF literals: $\{not\ a, not\ \neg a, not\ c, not\ \neg c, not\ d, not\ \neg d, not\ e, not\ \neg e\}$

ASP and Argumentation

Example (Answer Set Programming)

$$
\begin{array}{lll}
a & \leftarrow & not\ \neg a \\
 & & \\
d & \leftarrow & not\ \neg a \\
e & \leftarrow &
\end{array}
$$

$$e \in S$$
$$d \in S?$$
$$S = \{e, d, a\}$$

Interaction from **classical literals** to **NAF literals**!

Example (Assumption-Based Argumentation (ABA))

- **rules** = Answer Set Program
- **assumptions** = NAF literals: $\{not\ a, not\ \neg a, not\ c, not\ \neg c, not\ d, not\ \neg d, not\ e, not\ \neg e\}$
- **contraries**: $\overline{not\ a} = a$; $\overline{not\ \neg a} = \neg a$; $\overline{not\ c} = c$; $\ldots$

ASP and Argumentation

Example (Assumption-Based Argumentation (ABA))

- **rules** = Answer Set Program
- **assumptions** = NAF literals: $\{not\ a, not\ \neg a, not\ c, not\ \neg c, not\ d, not\ \neg d, not\ e, not\ \neg e\}$
- **contraries**: $\overline{not\ a} = a$; $\overline{not\ \neg a} = \neg a$; $\overline{not\ c} = c$; $\ldots$

semantics:

Example (Assumption-Based Argumentation (ABA))

- **rules** = Answer Set Program
- **assumptions** = NAF literals: $\{not\ a, not\ \neg a, not\ c, not\ \neg c, not\ d, not\ \neg d, not\ e, not\ \neg e\}$
- **contraries**: $\overline{not\ a} = a$; $\overline{not\ \neg a} = \neg a$; $\overline{not\ c} = c$; ...

semantics:

- construct arguments

ASP and Argumentation

Example (Assumption-Based Argumentation (ABA))

- **rules** = Answer Set Program
- **assumptions** = NAF literals: $\{not\ a, not\ \neg a, not\ c, not\ \neg c, not\ d, not\ \neg d, not\ e, not\ \neg e\}$
- **contraries**: $\overline{not\ a} = a$; $\overline{not\ \neg a} = \neg a$; $\overline{not\ c} = c$; $\ldots$

semantics:

- construct arguments
- attacks between arguments

ASP and Argumentation

Example (Assumption-Based Argumentation (ABA))

- **rules** = Answer Set Program
- **assumptions** = NAF literals: $\{not\ a, not\ \neg a, not\ c, not\ \neg c, not\ d, not\ \neg d, not\ e, not\ \neg e\}$
- **contraries**: $\overline{not\ a} = a$; $\overline{not\ \neg a} = \neg a$; $\overline{not\ c} = c$; $\ldots$

semantics:

- construct arguments
- attacks between arguments
- extensions = arguments "winning" together

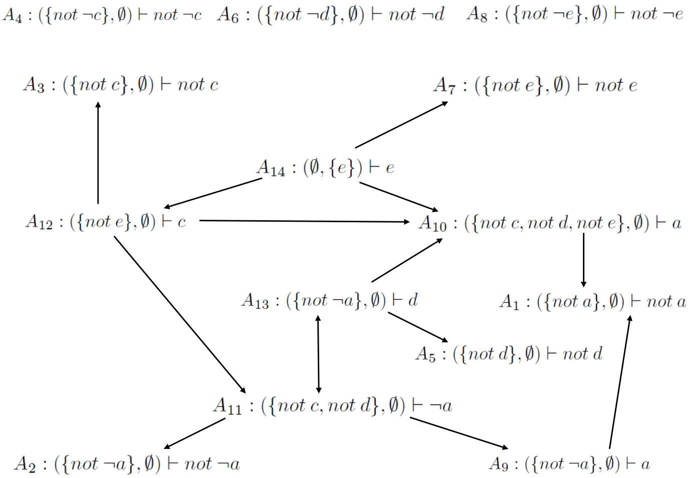
Example (Assumption-Based Argumentation (ABA))

- **rules** = Answer Set Program
- **assumptions** = NAF literals: $\{not\ a, not\ \neg a, not\ c, not\ \neg c, not\ d, not\ \neg d, not\ e, not\ \neg e\}$
- **contraries**: $\overline{not\ a} = a$; $\overline{not\ \neg a} = \neg a$; $\overline{not\ c} = c$; $\ldots$

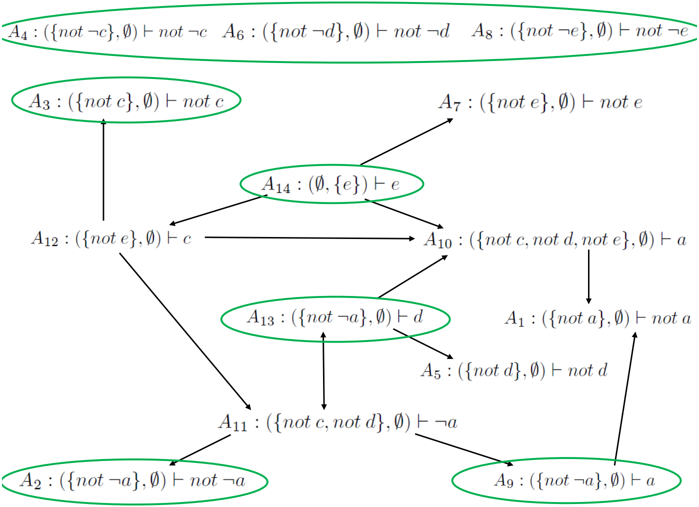
semantics:

- construct arguments
- attacks between arguments
- extensions = arguments "winning" together
  $\Rightarrow$ human-like reasoning

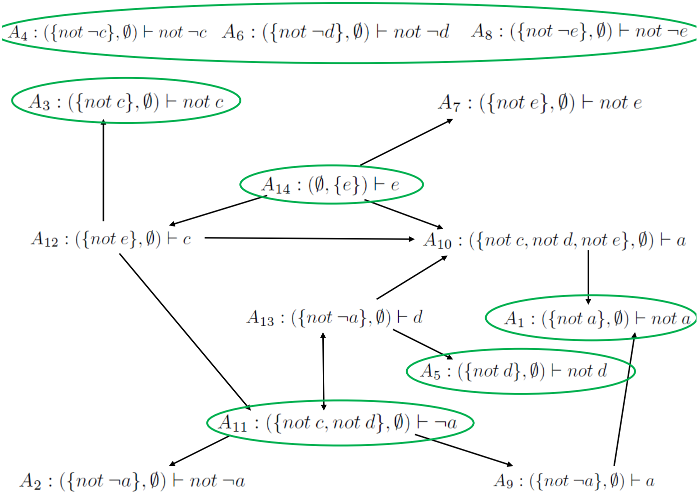
ABA semantics



ABA semantics



ABA semantics



ASP and Argumentation

Example (Assumption-Based Argumentation (ABA))

- **rules** = Answer Set Program
- **assumptions** = NAF literals: $\{not\ a, not\ \neg a, not\ c, not\ \neg c, not\ d, not\ \neg d, not\ e, not\ \neg e\}$
- **contraries**: $\overline{not\ a} = a$; $\overline{not\ \neg a} = \neg a$; $\overline{not\ c} = c$; ...

semantics:

- construct arguments
- attacks between arguments
- extensions = arguments "winning" together
  $\Rightarrow$ human-like reasoning

ASP and Argumentation

Example (Assumption-Based Argumentation (ABA))

- **rules** = Answer Set Program
- **assumptions** = NAF literals: $\{not\ a, not\ \neg a, not\ c, not\ \neg c,$ $not\ d, not\ \neg d, not\ e, not\ \neg e\}$
- **contraries**: $\overline{not\ a} = a$; $\overline{not\ \neg a} = \neg a$; $\overline{not\ c} = c$; $\ldots$

semantics:

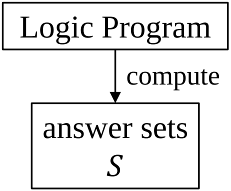
- construct arguments
- attacks between arguments
- extensions = arguments "winning" together
  $\Rightarrow$ human-like reasoning

extensions and answer sets correspond!

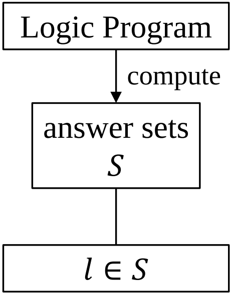
ABA-Based Answer Set Justifications - an overview

$\boxed{\text{Logic Program}}$

ABA-Based Answer Set Justifications - an overview



ABA-Based Answer Set Justifications - an overview



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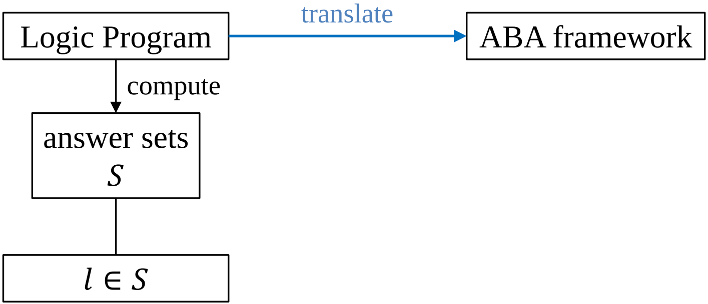

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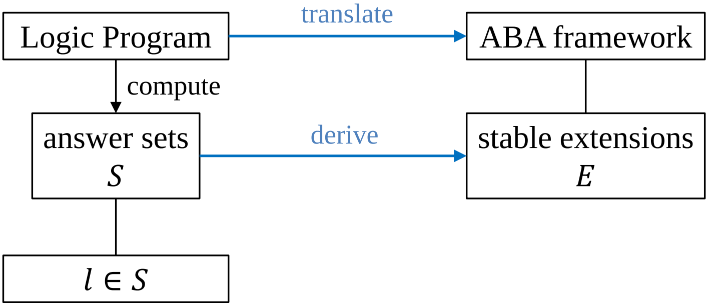

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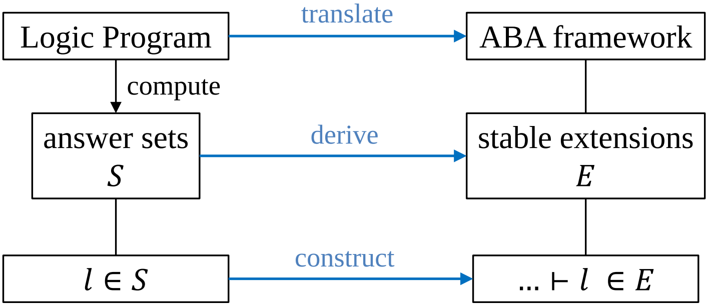
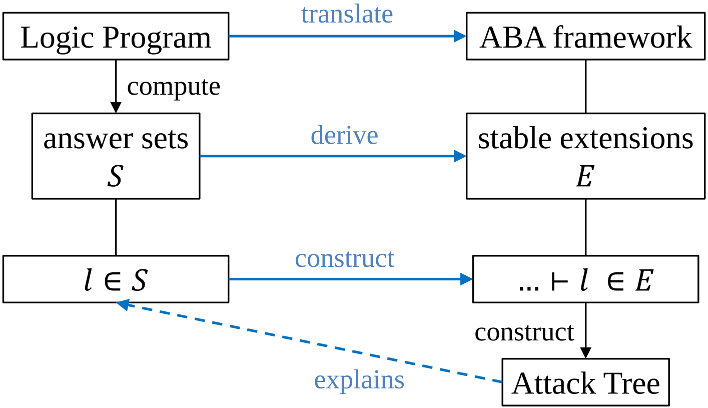


ABA-Based Answer Set Justifications - an overview



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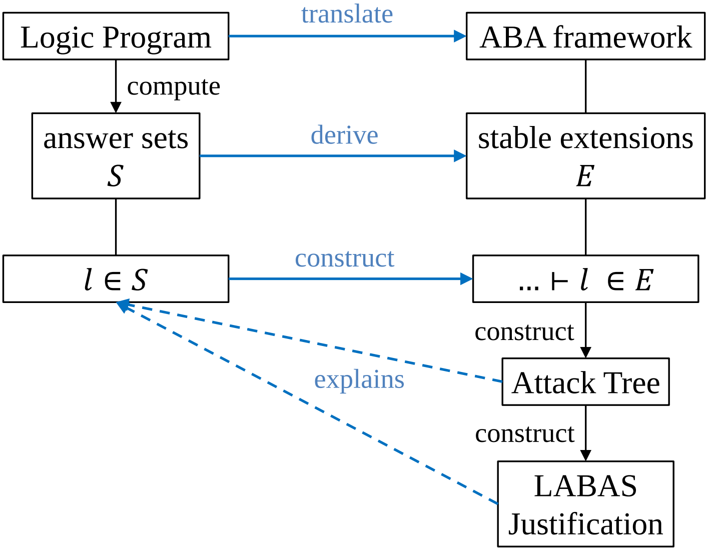
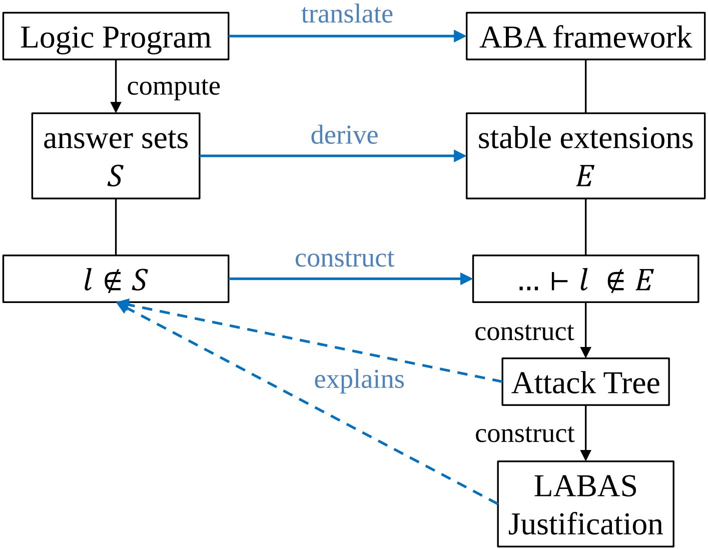
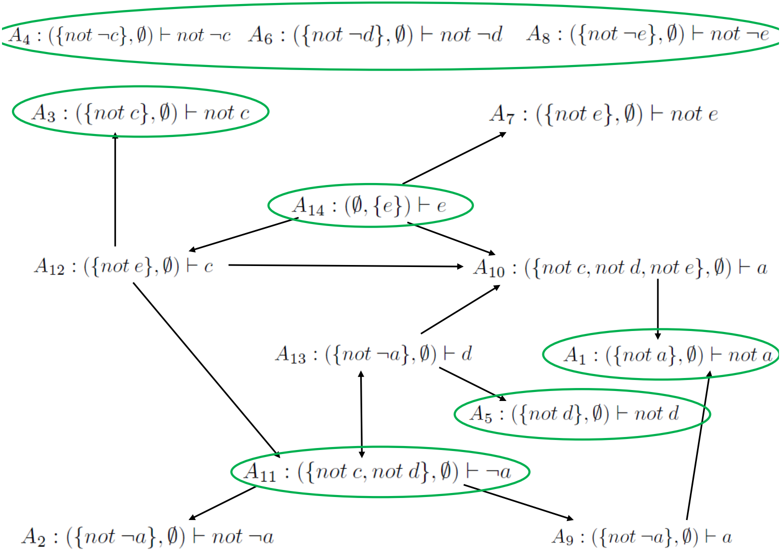


ABA-Based Answer Set Justifications - an overview



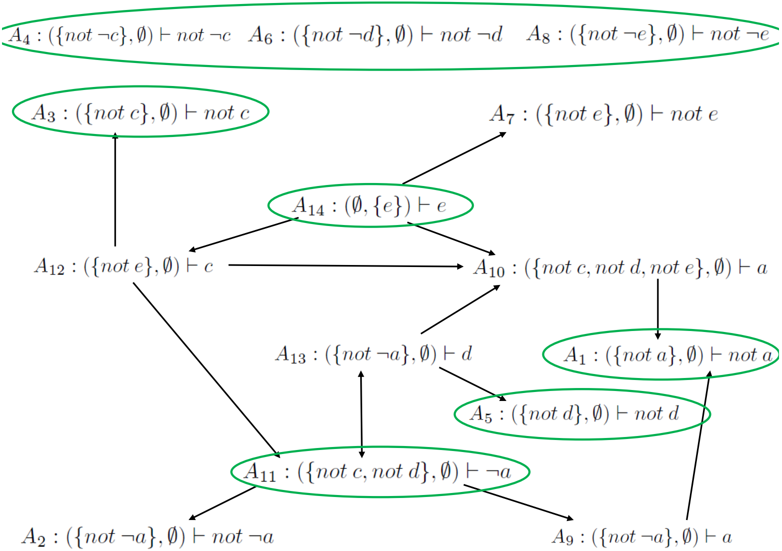
Attack Trees



Attack Trees

$$A_9^- : (\{not\ \neg a\}, \emptyset) \vdash a$$

Attack Trees



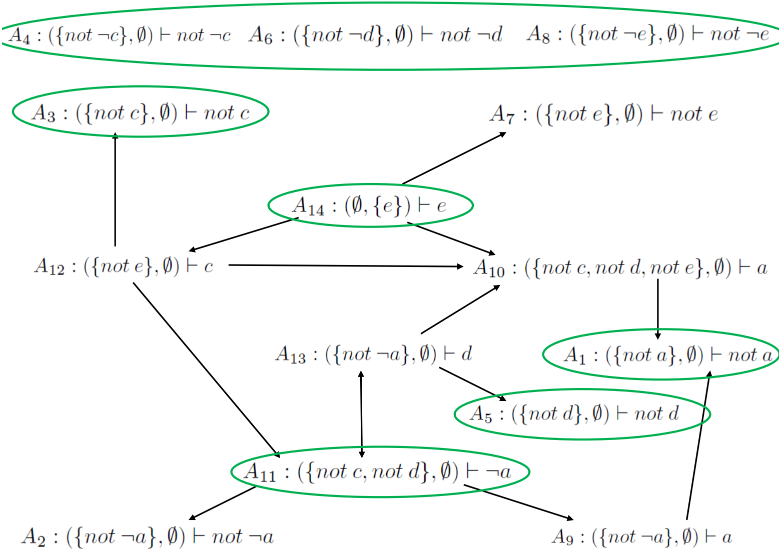
Attack Trees

$$A_9^- : (\{not\ \neg a\}, \emptyset) \vdash a$$

$$\uparrow$$

$$A_{11}^+ : (\{not\ c, not\ d\}, \emptyset) \vdash \neg a$$

Attack Trees



Attack Trees

$$A_9^- : (\{not\ \neg a\}, \emptyset) \vdash a$$

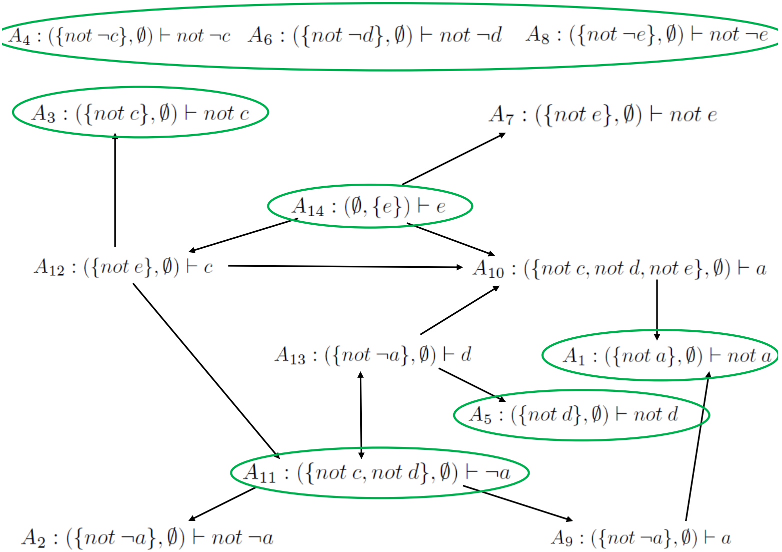
$$\uparrow$$

$$A_{11}^+ : (\{not\ c, not\ d\}, \emptyset) \vdash \neg a$$

$$\nearrow \qquad\qquad \nwarrow$$

$$A_{12}^- : (\{not\ e\}, \emptyset) \vdash c \qquad\qquad A_{13}^- : (\{not\ \neg a\}, \emptyset) \vdash d$$

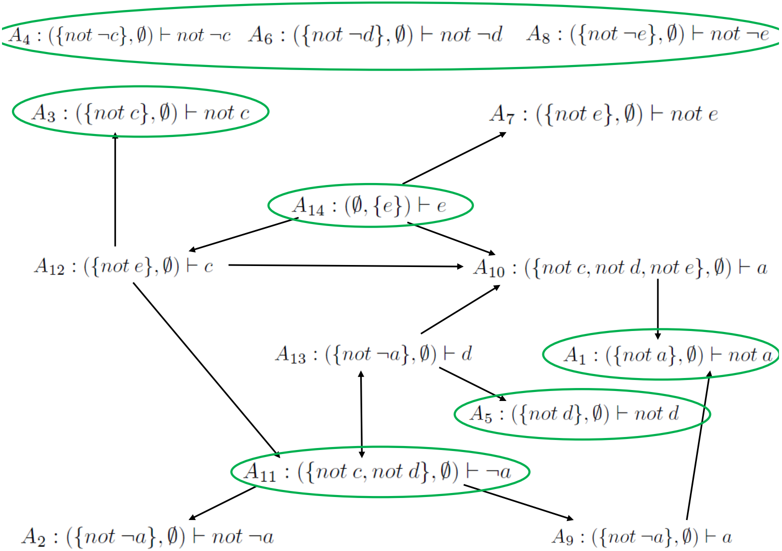
Attack Trees



Attack Trees

$$
\begin{tikzcd}
 & A_9^- : (\{not\ \neg a\}, \emptyset) \vdash a & \\
 & A_{11}^+ : (\{not\ c, not\ d\}, \emptyset) \vdash \neg a \arrow[u] & \\
A_{12}^- : (\{not\ e\}, \emptyset) \vdash c \arrow[ur] & & A_{13}^- : (\{not\ \neg a\}, \emptyset) \vdash d \arrow[ul] \\
A_{14}^+ : (\emptyset, \{e\}) \vdash e \arrow[u] & &
\end{tikzcd}
$$

Attack Trees



Attack Trees

$$
\begin{tikzcd}[column sep=small]
& A_9^- : (\{not\ \neg a\}, \emptyset) \vdash a & \\
& A_{11}^+ : (\{not\ c, not\ d\}, \emptyset) \vdash \neg a \arrow[u] & \\
A_{12}^- : (\{not\ e\}, \emptyset) \vdash c \arrow[ur] & & A_{13}^- : (\{not\ \neg a\}, \emptyset) \vdash d \arrow[ul] \\
A_{14}^+ : (\emptyset, \{e\}) \vdash e \arrow[u] & & \color{blue}{A_{11}^+ : (\{not\ c, not\ d\}, \emptyset) \vdash \neg a} \arrow[u, blue]
\end{tikzcd}
$$

Attack Trees

$$
\begin{tikzcd}[column sep=tiny]
 & A_9^- : (\{not\ \neg a\}, \emptyset) \vdash a & & \\
 & A_{11}^+ : (\{not\ c, not\ d\}, \emptyset) \vdash \neg a \arrow[u] & & \\
A_{12}^- : (\{not\ e\}, \emptyset) \vdash c \arrow[ur] & & A_{13}^- : (\{not\ \neg a\}, \emptyset) \vdash d \arrow[ul] & \\
A_{14}^+ : (\emptyset, \{e\}) \vdash e \arrow[u] & & A_{11}^+ : (\{not\ c, not\ d\}, \emptyset) \vdash \neg a \arrow[u] & \\
 & A_{12}^- : (\{not\ e\}, \emptyset) \vdash c \arrow[ur] & & A_{13}^- : (\{not\ \neg a\}, \emptyset) \vdash d \arrow[ul] \\
 & A_{14}^+ : (\emptyset, \{e\}) \vdash e \arrow[u] & & \vdots
\end{tikzcd}
$$

laser surgery!

Answer Set Programming (ASP)

intraocular lenses!

Why is **laser surgery** not part of the solution?

Answer Set:

$\{shortSighted, afraidToTouchEyes, student, likesSports, tightOnMoney, correctiveLens, caresAboutPracticality, intraocularLens\}$

Why is **laser surgery** not part of the solution?

Answer Set:

$\{shortSighted, afraidToTouchEyes, student, likesSports,$
$tightOnMoney, correctiveLens, caresAboutPracticality, intraocularLens\}$

$${A_1}^- : (\{shortSighted\}, \{not\ tightOnMoney, not\ correctiveLens\}) \vdash laserSurgery$$

$$\uparrow$$

$${A_2}^+ : (\{student\}, \{not\ richParents\}) \vdash tightOnMoney$$

Why is **intraocular lens** part of the solution?

Answer Set:

$\{shortSighted, afraidToTouchEyes, student, likesSports, tightOnMoney, correctiveLens, caresAboutPracticality, intraocularLens\}$

Why is **intraocular lens** part of the solution?

Answer Set:

$\{shortSighted, afraidToTouchEyes, student, likesSports,$
$tightOnMoney, correctiveLens, caresAboutPracticality, intraocularLens\}$

$A_3{}^{+} : (\{shortSighted\}, \{not\ laserSurgery, not\ glasses, not\ contactLens\}) \vdash intraocularLens$

$A_1{}^{-} : (\ldots) \vdash laserSurgery$

$A_4^{-} : (\ldots, not\ afraidToTouchEyes, \ldots) \vdash contactLens$

$A_2{}^{+} : (\ldots) \vdash tightOnMoney$

$A_5^{+} : (\{afraidToTouchEyes\}, \emptyset) \vdash afraidToTouchEyes$

$A_6^{-} : (\ldots, not\ caresAboutPracticality, \ldots) \vdash glasses$

$A_7^{+} : (\{likesSports\}, \emptyset) \vdash caresAboutPracticality$

Labelled ABA-Based Answer Set (LABAS) Justifications

$$A_9^- : (\{not\ \neg a\}, \emptyset) \vdash a$$

$$A_{11}^+ : (\{not\ c, not\ d\}, \emptyset) \vdash \neg a$$

$$A_{12}^- : (\{not\ e\}, \emptyset) \vdash c \qquad A_{13}^- : (\{not\ \neg a\}, \emptyset) \vdash d$$

$$A_{14}^+ : (\emptyset, \{e\}) \vdash e \qquad A_{11}^+ : (\{not\ c, not\ d\}, \emptyset) \vdash \neg a$$

$$A_{12}^- : (\{not\ e\}, \emptyset) \vdash c \qquad A_{13}^- : (\{not\ \neg a\}, \emptyset) \vdash d$$

$$A_{14}^+ : (\emptyset, \{e\}) \vdash e \qquad \vdots$$

Labelled ABA-Based Answer Set (LABAS) Justifications

$a^{-}_{A_9}$

Labelled ABA-Based Answer Set (LABAS) Justifications

$$
\begin{tikzcd}
& A_9^- : (\{not\ \neg a\}, \emptyset) \vdash a & & \\
& A_{11}^+ : (\{not\ c, not\ d\}, \emptyset) \vdash \neg a \arrow[u] & & \\
A_{12}^- : (\{not\ e\}, \emptyset) \vdash c \arrow[ur] & & A_{13}^- : (\{not\ \neg a\}, \emptyset) \vdash d \arrow[ul] & \\
A_{14}^+ : (\emptyset, \{e\}) \vdash e \arrow[u] & & A_{11}^+ : (\{not\ c, not\ d\}, \emptyset) \vdash \neg a \arrow[u] & \\
& A_{12}^- : (\{not\ e\}, \emptyset) \vdash c \arrow[ur] & & A_{13}^- : (\{not\ \neg a\}, \emptyset) \vdash d \arrow[ul] \\
& A_{14}^+ : (\emptyset, \{e\}) \vdash e \arrow[u] & & \vdots
\end{tikzcd}
$$

Labelled ABA-Based Answer Set (LABAS) Justifications

$$\begin{tikzcd}
a^{-}_{A_9} \\
\mathit{not}\ \neg a^{-}_{asm} \arrow[u, dashed, "-"]
\end{tikzcd}$$

Labelled ABA-Based Answer Set (LABAS) Justifications

$$\begin{tikzcd}
 & A_9^- : (\{not\ \neg a\}, \emptyset) \vdash a & & \\
 & A_{11}^+ : (\{not\ c, not\ d\}, \emptyset) \vdash \neg a \arrow[u] & & \\
A_{12}^- : (\{not\ e\}, \emptyset) \vdash c \arrow[ur] & & A_{13}^- : (\{not\ \neg a\}, \emptyset) \vdash d \arrow[ul] & \\
A_{14}^+ : (\emptyset, \{e\}) \vdash e \arrow[u] & & A_{11}^+ : (\{not\ c, not\ d\}, \emptyset) \vdash \neg a \arrow[u] & \\
 & A_{12}^- : (\{not\ e\}, \emptyset) \vdash c \arrow[ur] & & A_{13}^- : (\{not\ \neg a\}, \emptyset) \vdash d \arrow[ul] \\
 & A_{14}^+ : (\emptyset, \{e\}) \vdash e \arrow[u] & & \vdots
\end{tikzcd}$$

Labelled ABA-Based Answer Set (LABAS) Justifications

$$\begin{tikzcd}
a^{-}_{A_9} \\
not\ \neg a^{-}_{asm} \arrow[u, dashed, "-"'] \\
\neg a^{+}_{A_{11}} \arrow[u, "+"']
\end{tikzcd}$$

Labelled ABA-Based Answer Set (LABAS) Justifications

$$
\begin{tikzcd}[column sep=tiny]
 & A_9^- : (\{not\ \neg a\}, \emptyset) \vdash a & & \\
 & A_{11}^+ : (\{not\ c, not\ d\}, \emptyset) \vdash \neg a \arrow[u] & & \\
A_{12}^- : (\{not\ e\}, \emptyset) \vdash c \arrow[ur] & & A_{13}^- : (\{not\ \neg a\}, \emptyset) \vdash d \arrow[ul] & \\
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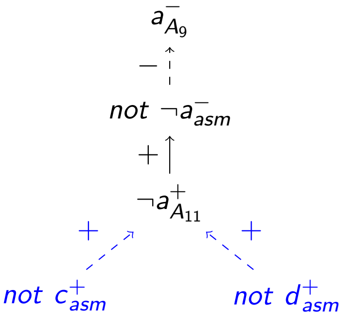
 & A_{14}^+ : (\emptyset, \{e\}) \vdash e \arrow[u] & & \vdots
\end{tikzcd}
$$

Labelled ABA-Based Answer Set (LABAS) Justifications



Labelled ABA-Based Answer Set (LABAS) Justifications

$$
\begin{tikzcd}
 & A_9^- : (\{not\ \neg a\}, \emptyset) \vdash a & & \\
 & A_{11}^+ : (\{not\ c, not\ d\}, \emptyset) \vdash \neg a \arrow[u] & & \\
A_{12}^- : (\{not\ e\}, \emptyset) \vdash c \arrow[ur] & & A_{13}^- : (\{not\ \neg a\}, \emptyset) \vdash d \arrow[ul] & \\
A_{14}^+ : (\emptyset, \{e\}) \vdash e \arrow[u] & & A_{11}^+ : (\{not\ c, not\ d\}, \emptyset) \vdash \neg a \arrow[u] & \\
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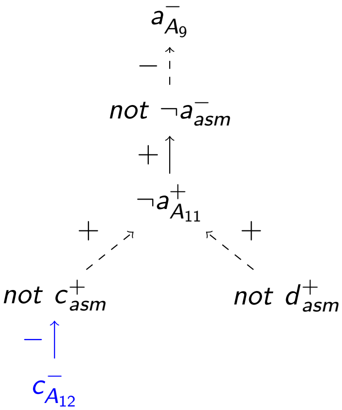
$$

Labelled ABA-Based Answer Set (LABAS) Justifications



Labelled ABA-Based Answer Set (LABAS) Justifications

$$
\begin{tikzcd}[column sep=tiny]
 & & A_9^- : (\{not\ \neg a\}, \emptyset) \vdash a & & \\
 & & A_{11}^+ : (\{not\ c, not\ d\}, \emptyset) \vdash \neg a \arrow[u] & & \\
 & A_{12}^- : (\{not\ e\}, \emptyset) \vdash c \arrow[ur] & & A_{13}^- : (\{not\ \neg a\}, \emptyset) \vdash d \arrow[ul] & \\
 & A_{14}^+ : (\emptyset, \{e\}) \vdash e \arrow[u] & & A_{11}^+ : (\{not\ c, not\ d\}, \emptyset) \vdash \neg a \arrow[u] & \\
 & & A_{12}^- : (\{not\ e\}, \emptyset) \vdash c \arrow[ur] & & A_{13}^- : (\{not\ \neg a\}, \emptyset) \vdash d \arrow[ul] \\
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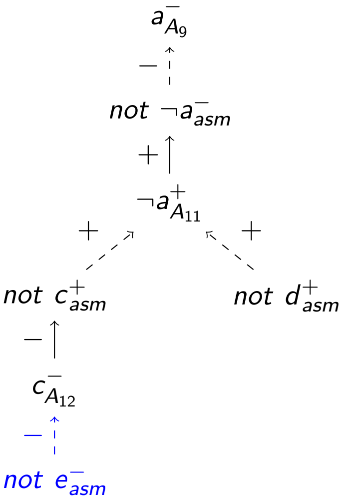
\end{tikzcd}
$$

Labelled ABA-Based Answer Set (LABAS) Justifications



Labelled ABA-Based Answer Set (LABAS) Justifications

$$
\begin{tikzcd}[column sep=tiny]
 & A_9^- : (\{not\ \neg a\}, \emptyset) \vdash a & & \\
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A_{12}^- : (\{not\ e\}, \emptyset) \vdash c \arrow[ur] & & A_{13}^- : (\{not\ \neg a\}, \emptyset) \vdash d \arrow[ul] & \\
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 & A_{12}^- : (\{not\ e\}, \emptyset) \vdash c \arrow[ur] & & A_{13}^- : (\{not\ \neg a\}, \emptyset) \vdash d \arrow[ul] \\
 & A_{14}^+ : (\emptyset, \{e\}) \vdash e \arrow[u] & & \vdots
\end{tikzcd}
$$

Labelled ABA-Based Answer Set (LABAS) Justifications

$$
\begin{tikzcd}
 & a^{-}_{A_9} & \\
 & not\ \neg a^{-}_{asm} \arrow[u, dashed, "-"] & \\
 & \neg a^{+}_{A_{11}} \arrow[u, "+"] & \\
not\ c^{+}_{asm} \arrow[ur, dashed, "+"] & & not\ d^{+}_{asm} \arrow[ul, dashed, "+"'] \\
c^{-}_{A_{12}} \arrow[u, "-"] & & \\
not\ e^{-}_{asm} \arrow[u, dashed, "-"] & & \\
\color{blue}{e^{+}_{fact}} \arrow[u, blue, "+"] & &
\end{tikzcd}
$$

Labelled ABA-Based Answer Set (LABAS) Justifications

$$
\begin{tikzcd}[column sep=tiny]
 & & A_9^- : (\{not\ \neg a\}, \emptyset) \vdash a & & \\
 & & A_{11}^+ : (\{not\ c, not\ d\}, \emptyset) \vdash \neg a \arrow[u] & & \\
 & A_{12}^- : (\{not\ e\}, \emptyset) \vdash c \arrow[ur] & & A_{13}^- : (\{not\ \neg a\}, \emptyset) \vdash d \arrow[ul] & \\
 & A_{14}^+ : (\emptyset, \{e\}) \vdash e \arrow[u] & & A_{11}^+ : (\{not\ c, not\ d\}, \emptyset) \vdash \neg a \arrow[u] & \\
 & & A_{12}^- : (\{not\ e\}, \emptyset) \vdash c \arrow[ur] & & A_{13}^- : (\{not\ \neg a\}, \emptyset) \vdash d \arrow[ul] \\
 & & A_{14}^+ : (\emptyset, \{e\}) \vdash e \arrow[u] & & \vdots
\end{tikzcd}
$$

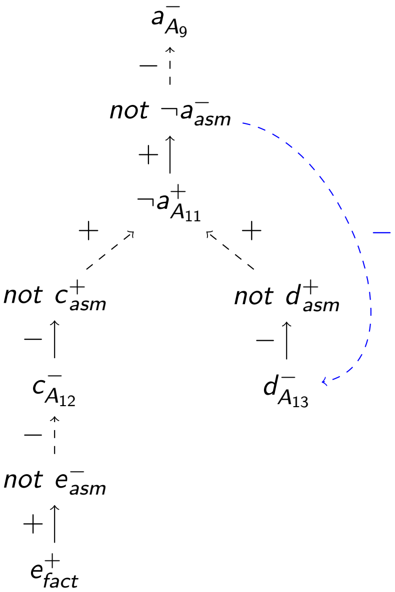
Labelled ABA-Based Answer Set (LABAS) Justifications

$$\begin{tikzcd}
 & a^{-}_{A_9} & \\
 & \mathit{not}\ \neg a^{-}_{asm} \arrow[u, dashed, "-"] & \\
 & \neg a^{+}_{A_{11}} \arrow[u, "+"] & \\
\mathit{not}\ c^{+}_{asm} \arrow[ur, dashed, "+"] & & \mathit{not}\ d^{+}_{asm} \arrow[ul, dashed, "+"'] \\
c^{-}_{A_{12}} \arrow[u, "-"] & & d^{-}_{A_{13}} \arrow[u, "-"] \\
\mathit{not}\ e^{-}_{asm} \arrow[u, dashed, "-"] & & \\
e^{+}_{fact} \arrow[u, "+"] & &
\end{tikzcd}$$

Labelled ABA-Based Answer Set (LABAS) Justifications

$$\begin{tikzcd}
 & A_9^- : (\{not\ \neg a\}, \emptyset) \vdash a & & \\
 & A_{11}^+ : (\{not\ c, not\ d\}, \emptyset) \vdash \neg a \arrow[u] & & \\
A_{12}^- : (\{not\ e\}, \emptyset) \vdash c \arrow[ur] & & A_{13}^- : (\{not\ \neg a\}, \emptyset) \vdash d \arrow[ul] & \\
A_{14}^+ : (\emptyset, \{e\}) \vdash e \arrow[u] & & A_{11}^+ : (\{not\ c, not\ d\}, \emptyset) \vdash \neg a \arrow[u] & \\
 & A_{12}^- : (\{not\ e\}, \emptyset) \vdash c \arrow[ur] & & A_{13}^- : (\{not\ \neg a\}, \emptyset) \vdash d \arrow[ul] \\
 & A_{14}^+ : (\emptyset, \{e\}) \vdash e \arrow[u] & & \vdots
\end{tikzcd}$$

Labelled ABA-Based Answer Set (LABAS) Justifications



laser surgery!

Answer Set Programming
(ASP)

intraocular lenses!

Why is **laser surgery** not part of the solution?

Attack Tree

$$A_1{}^{-} : (\{shortSighted\}, \{not\ tightOnMoney, not\ correctiveLens\}) \vdash laserSurgery$$

$$\uparrow$$

$$A_2{}^{+} : (\{student\}, \{not\ richParents\}) \vdash tightOnMoney$$

Why is **laser surgery** not part of the solution?

Attack Tree

$$\begin{tikzcd}
{A_1}^{-} : (\{\mathit{shortSighted}\}, \{\mathit{not\ tightOnMoney}, \mathit{not\ correctiveLens}\}) \vdash \mathit{laserSurgery} \\
{A_2}^{+} : (\{\mathit{student}\}, \{\mathit{not\ richParents}\}) \vdash \mathit{tightOnMoney} \arrow[u]
\end{tikzcd}$$

ABA-Based Answer Set (ABAS) Justification

$$\begin{tikzcd}
& \mathit{laserSurgery}^{-}_{A_1} & \\
& \mathit{not\ tightOnMoney}^{-}_{asm} \arrow[u, dashed, "-"] & \\
& \mathit{tightOnMoney}^{+}_{A_2} \arrow[u, "+"] & \\
\mathit{student}^{+}_{fact} \arrow[ur, dashed, "+"] & & \mathit{not\ richParents}^{+}_{asm} \arrow[ul, dashed, "+"']
\end{tikzcd}$$

Why is **intraocular lens** part of the solution?

Attack Tree

$A_3{}^+ : (\{\mathit{shortSighted}\}, \{\mathit{not\ laserSurgery}, \mathit{not\ glasses}, \mathit{not\ contactLens}\}) \vdash \mathit{intraocularLens}$

$A_1{}^- : (\ldots) \vdash \mathit{laserSurgery}$

$A_4^- : (\ldots, \mathit{not\ afraidToTouchEyes}, \ldots) \vdash \mathit{contactLens}$

$A_2{}^+ : (\ldots) \vdash \mathit{tightOnMoney}$

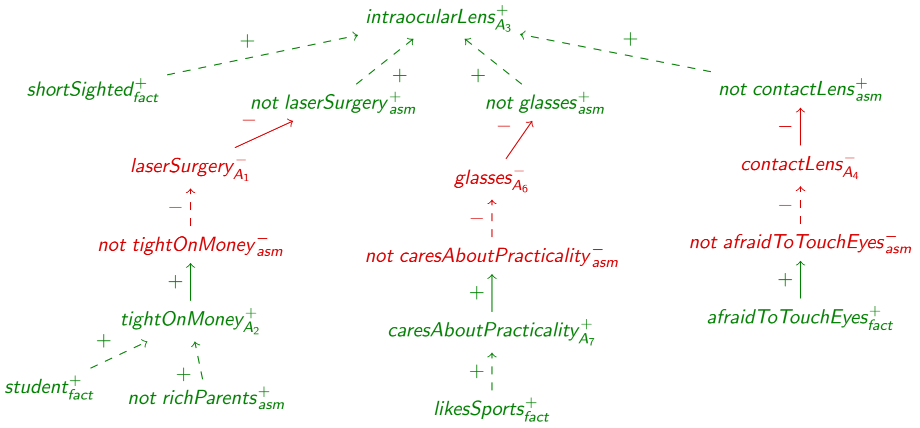
$A_5^+ : (\{\mathit{afraidToTouchEyes}\}, \emptyset) \vdash \mathit{afraidToTouchEyes}$

$A_6^- : (\ldots, \mathit{not\ caresAboutPracticality}, \ldots) \vdash \mathit{glasses}$

$A_7^+ : (\{\mathit{likesSports}\}, \emptyset) \vdash \mathit{caresAboutPracticality}$

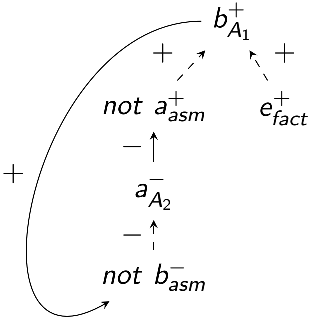
Why is **intraocular lens** part of the solution?

ABA-Based Answer Set (ABAS) Justification

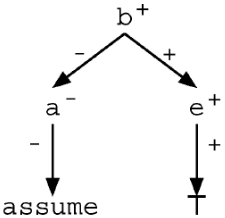


Other justificiation approaches

LABAS Justification

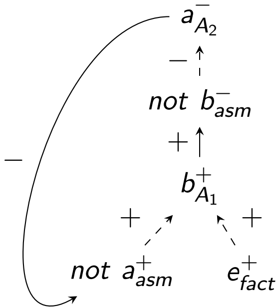


Off-line Justification (Pontelli, Son, Elkhatib)

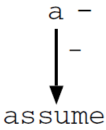


Other justificiation approaches

LABAS Justification



Off-line Justification (Pontelli, Son, Elkhatib)



Conclusion

Answer Set Programming (ASP)

$+$

ABA-Based Answer Set Justification

$=$

Conclusion

Answer Set Programming (ASP)

$$+$$

ABA-Based Answer Set Justification

$$=$$

~~laser surgery!~~

intraocular lenses!

Future Work

So far: restricted do **consistent** logic programs

Future Work

So far: restricted do **consistent** logic programs

- find source of **inconsistency** in a logic program
- **debug** the logic program

$\Rightarrow$ more existing literature

IMPERIAL COLLEGE COMPUTER STUDENT WORKSHOP 2015

24-25 September 2015
London, United Kingdom

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The workshop offers:

- A traditional track featuring
  - Technical papers focused on current topics in computer science
  - Tool papers describing the design, functionality and applicability of a software tool
- A survey track featuring
  - Papers that synthesise the state of the art in a particular topic of computer science

Areas of interest cover all fields of research in computer science, including (but not limited to) the following:

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- Theory of Computational Systems
- Verification and Model Checking
- Machine Learning
- Computer Systems
- Networks and Communications
- Logicand Artificial Intelligence
- Programming Languages
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- Databases

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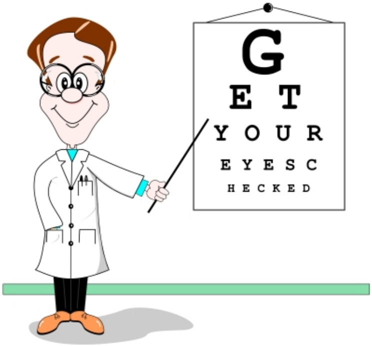
iccsw@imperial.ac.uk

ICCSW 15



Questions?!

ASP Semantics

Why not simply display the derivation?

The **answer set** of $\mathcal{P}$ ($\mathcal{AS}(\mathcal{P})$), is the smallest set $S \subseteq Lit_{\mathcal{P}}$ s.t.:

1. for any clause $l_0 \leftarrow l_1, \ldots, l_m$ in $\mathcal{P}$:
   if $l_1, \ldots, l_m \in S$ then $l_0 \in S$
2. $S = Lit_{\mathcal{P}}$ if S contains complementary literals a and $\neg a$.

$\Rightarrow$ For $\mathcal{P}$ without NAF literals

For $\mathcal{P}$ with NAF literals

S is an **answer set** of $\mathcal{P}$ if it is the answer set of the reduct $\mathcal{P}^S$, i.e. if $S = \mathcal{AS}(\mathcal{P}^S)$.

ASP Semantics

It all depends on the reduct...

For $\mathcal{P}$ possibly with NAF literals and for any $S \subseteq Lit_{\mathcal{P}}$

The **reduct** $\mathcal{P}^S$ is obtained from $\mathcal{P}$ by deleting:

1. all clauses with *not* l in their bodies where $l \in S$
2. all NAF literals in the remaining clauses.

ASP Semantics

It all depends on the reduct...

For $\mathcal{P}$ possibly with NAF literals and for any $S \subseteq Lit_{\mathcal{P}}$

The **reduct** $\mathcal{P}^S$ is obtained from $\mathcal{P}$ by deleting:

1. all clauses with *not* l in their bodies where $l \in S$
2. all NAF literals in the remaining clauses.

Example

$$
\begin{array}{lcl}
a & \leftarrow & not\ \neg a \\
a & \leftarrow & \neg a, not\ c, not\ e \\
\neg a & \leftarrow & not\ c, not\ d \\
c & \leftarrow & not\ e \\
d & \leftarrow & not\ \neg a \\
e & \leftarrow &
\end{array}
$$

ASP Semantics

It all depends on the reduct...

For $\mathcal{P}$ possibly with NAF literals and for any $S \subseteq Lit_{\mathcal{P}}$

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e & \leftarrow &
\end{array}
$$

$e \in S$

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Example

$$
\begin{array}{rcl}
a & \leftarrow & not\ \neg a \\
a & \leftarrow & \neg a, not\ c, not\ e \\
\neg a & \leftarrow & not\ c, not\ d \\
c & \leftarrow & not\ e \\
d & \leftarrow & not\ \neg a \\
e & \leftarrow &
\end{array}
$$

$e \in S$

ASP Semantics

It all depends on the reduct...

For $\mathcal{P}$ possibly with NAF literals and for any $S \subseteq Lit_{\mathcal{P}}$

The **reduct** $\mathcal{P}^S$ is obtained from $\mathcal{P}$ by deleting:

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Example

$$
\begin{array}{lcl}
a & \leftarrow & not\ \neg a \\
\neg a & \leftarrow & not\ c, not\ d \\
d & \leftarrow & not\ \neg a \\
e & \leftarrow &
\end{array}
$$

$e \in S$

ASP Semantics

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For $\mathcal{P}$ possibly with NAF literals and for any $S \subseteq Lit_{\mathcal{P}}$

The **reduct** $\mathcal{P}^S$ is obtained from $\mathcal{P}$ by deleting:

1. all clauses with *not* l in their bodies where $l \in S$
2. all NAF literals in the remaining clauses.

Example

$$\begin{array}{rcl} a & \leftarrow & not\ \neg a \\ \neg a & \leftarrow & not\ c, not\ d \\ d & \leftarrow & not\ \neg a \\ e & \leftarrow & \end{array}$$

$e \in S$

$d \in S$?

ASP Semantics

It all depends on the reduct...

For $\mathcal{P}$ possibly with NAF literals and for any $S \subseteq Lit_{\mathcal{P}}$

The **reduct** $\mathcal{P}^S$ is obtained from $\mathcal{P}$ by deleting:

1. all clauses with *not* l in their bodies where $l \in S$
2. all NAF literals in the remaining clauses.

Example

$$
\begin{array}{lll}
a & \leftarrow & not\ \neg a \\
\neg a & \leftarrow & not\ c,\ not\ d \\
d & \leftarrow & not\ \neg a \\
e & \leftarrow &
\end{array}
\qquad
\begin{array}{c}
e \in S \\
d \in S?
\end{array}
$$

ASP Semantics

It all depends on the reduct...

For $\mathcal{P}$ possibly with NAF literals and for any $S \subseteq Lit_{\mathcal{P}}$

The **reduct** $\mathcal{P}^S$ is obtained from $\mathcal{P}$ by deleting:

1. all clauses with *not* l in their bodies where $l \in S$
2. all NAF literals in the remaining clauses.

Example

$$
\begin{array}{lll}
a & \leftarrow & not\ \neg a \\
 & & \\
d & \leftarrow & not\ \neg a \\
e & \leftarrow &
\end{array}
$$

$e \in S$

$d \in S$?

ASP Semantics

It all depends on the reduct...

For $\mathcal{P}$ possibly with NAF literals and for any $S \subseteq Lit_{\mathcal{P}}$

The **reduct** $\mathcal{P}^S$ is obtained from $\mathcal{P}$ by deleting:

1. all clauses with *not* l in their bodies where $l \in S$
2. all NAF literals in the remaining clauses.

Example

$$\begin{array}{lll} a & \leftarrow & not\ \neg a \\ & & \\ d & \leftarrow & not\ \neg a \\ e & \leftarrow & \end{array}$$

$$e \in S$$
$$d \in S?$$
$$S = \{e, d, a\}$$

ABA - formally

Example

$\mathcal{P}$:

$$
\begin{array}{lcl}
a & \leftarrow & not\ \neg a \\
a & \leftarrow & \neg a, not\ c, not\ e \\
\neg a & \leftarrow & not\ c, not\ d \\
c & \leftarrow & not\ e \\
d & \leftarrow & not\ \neg a \\
e & \leftarrow &
\end{array}
$$

$ABA_{\mathcal{P}} = \langle \mathcal{L}_{\mathcal{P}}, \mathcal{R}_{\mathcal{P}}, \mathcal{A}_{\mathcal{P}}, \bar{\ } \rangle$:

- **language**: $\mathcal{L}_{\mathcal{P}} = Lit_{\mathcal{P}} \cup NAF_{\mathcal{P}}$
- **rules**: $\mathcal{R}_{\mathcal{P}} = \mathcal{P}$
- **assumptions**: $\mathcal{A}_{\mathcal{P}} = NAF_{\mathcal{P}} = \{not\ a, not\ \neg a, not\ c, not\ \neg c, not\ d, not\ \neg d, not\ e, not\ \neg e\}$

ABA - formally

Example

$\mathcal{P}$:

$$
\begin{array}{lll}
a & \leftarrow & not\ \neg a \\
a & \leftarrow & \neg a, not\ c, not\ e \\
\neg a & \leftarrow & not\ c, not\ d \\
c & \leftarrow & not\ e \\
d & \leftarrow & not\ \neg a \\
e & \leftarrow &
\end{array}
$$

$ABA_{\mathcal{P}} = \langle \mathcal{L}_{\mathcal{P}}, \mathcal{R}_{\mathcal{P}}, \mathcal{A}_{\mathcal{P}}, \bar{\ } \rangle$:

- **language**: $\mathcal{L}_{\mathcal{P}} = Lit_{\mathcal{P}} \cup NAF_{\mathcal{P}}$
- **rules**: $\mathcal{R}_{\mathcal{P}} = \mathcal{P}$
- **assumptions**: $\mathcal{A}_{\mathcal{P}} = NAF_{\mathcal{P}} = \{not\ a, not\ \neg a, not\ c, not\ \neg c, not\ d, not\ \neg d, not\ e, not\ \neg e\}$
- **contraries**:
  $\overline{not\ a} = a$; $\overline{not\ \neg a} = \neg a$;
  $\overline{not\ c} = c$; $\overline{not\ \neg c} = \neg c$;
  $\overline{not\ d} = d$; $\overline{not\ \neg d} = \neg d$;
  $\overline{not\ e} = e$; $\overline{not\ \neg e} = \neg e$

argument: derivation (modus ponens) from assumptions and rules $(\{assumptions\}, \{facts\}) \vdash conclusion$

ABA - formally

Example

$\mathcal{P}$:

$$\begin{array}{lcl} a & \leftarrow & not\ \neg a \\ a & \leftarrow & \neg a, not\ c, not\ e \\ \neg a & \leftarrow & not\ c, not\ d \\ c & \leftarrow & not\ e \\ d & \leftarrow & not\ \neg a \\ e & \leftarrow & \end{array}$$

$ABA_{\mathcal{P}} = \langle \mathcal{L}_{\mathcal{P}}, \mathcal{R}_{\mathcal{P}}, \mathcal{A}_{\mathcal{P}}, \bar{\ } \rangle$:

- **language**: $\mathcal{L}_{\mathcal{P}} = Lit_{\mathcal{P}} \cup NAF_{\mathcal{P}}$
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- **assumptions**: $\mathcal{A}_{\mathcal{P}} = NAF_{\mathcal{P}} = \{not\ a, not\ \neg a, not\ c, not\ \neg c, not\ d, not\ \neg d, not\ e, not\ \neg e\}$
- **contraries**:
  $\overline{not\ a} = a$; $\overline{not\ \neg a} = \neg a$;
  $\overline{not\ c} = c$; $\overline{not\ \neg c} = \neg c$;
  $\overline{not\ d} = d$; $\overline{not\ \neg d} = \neg d$;
  $\overline{not\ e} = e$; $\overline{not\ \neg e} = \neg e$

argument: derivation (modus ponens) from assumptions and rules
$A_1 : (\{not\ a\}, \emptyset) \vdash not\ a$

ABA - formally

Example

$\mathcal{P}$:

$$
\begin{array}{lcl}
a & \leftarrow & not\ \neg a \\
a & \leftarrow & \neg a, not\ c, not\ e \\
\neg a & \leftarrow & not\ c, not\ d \\
c & \leftarrow & not\ e \\
d & \leftarrow & not\ \neg a \\
e & \leftarrow &
\end{array}
$$

$ABA_{\mathcal{P}} = \langle \mathcal{L}_{\mathcal{P}}, \mathcal{R}_{\mathcal{P}}, \mathcal{A}_{\mathcal{P}}, \bar{\ } \rangle$:

- **language**: $\mathcal{L}_{\mathcal{P}} = Lit_{\mathcal{P}} \cup NAF_{\mathcal{P}}$
- **rules**: $\mathcal{R}_{\mathcal{P}} = \mathcal{P}$
- **assumptions**: $\mathcal{A}_{\mathcal{P}} = NAF_{\mathcal{P}} = \{not\ a, not\ \neg a, not\ c, not\ \neg c, not\ d, not\ \neg d, not\ e, not\ \neg e\}$
- **contraries**:
  $\overline{not\ a} = a$; $\overline{not\ \neg a} = \neg a$;
  $\overline{not\ c} = c$; $\overline{not\ \neg c} = \neg c$;
  $\overline{not\ d} = d$; $\overline{not\ \neg d} = \neg d$;
  $\overline{not\ e} = e$; $\overline{not\ \neg e} = \neg e$

argument: derivation (modus ponens) from assumptions and rules

$A_{14} : (\emptyset, \{e\}) \vdash e$

ABA - formally

Example

$\mathcal{P}$:

$$
\begin{array}{lll}
a & \leftarrow & not\ \neg a \\
a & \leftarrow & \neg a, not\ c, not\ e \\
\neg a & \leftarrow & not\ c, not\ d \\
c & \leftarrow & not\ e \\
d & \leftarrow & not\ \neg a \\
e & \leftarrow &
\end{array}
$$

$ABA_{\mathcal{P}} = \langle \mathcal{L}_{\mathcal{P}}, \mathcal{R}_{\mathcal{P}}, \mathcal{A}_{\mathcal{P}}, \bar{\ } \rangle$:

- **language**: $\mathcal{L}_{\mathcal{P}} = Lit_{\mathcal{P}} \cup NAF_{\mathcal{P}}$
- **rules**: $\mathcal{R}_{\mathcal{P}} = \mathcal{P}$
- **assumptions**: $\mathcal{A}_{\mathcal{P}} = NAF_{\mathcal{P}} = \{not\ a, not\ \neg a, not\ c, not\ \neg c, not\ d, not\ \neg d, not\ e, not\ \neg e\}$
- **contraries**:
  $\overline{not\ a} = a$; $\overline{not\ \neg a} = \neg a$;
  $\overline{not\ c} = c$; $\overline{not\ \neg c} = \neg c$;
  $\overline{not\ d} = d$; $\overline{not\ \neg d} = \neg d$;
  $\overline{not\ e} = e$; $\overline{not\ \neg e} = \neg e$

argument: derivation (modus ponens) from assumptions and rules
$A_{13} : (\{not\ \neg a\}, \emptyset) \vdash d$

ABA - formally

Example

$\mathcal{P}$:

$$
\begin{array}{lll}
a & \leftarrow & not\ \neg a \\
a & \leftarrow & \neg a, not\ c, not\ e \\
\neg a & \leftarrow & not\ c, not\ d \\
c & \leftarrow & not\ e \\
d & \leftarrow & not\ \neg a \\
e & \leftarrow &
\end{array}
$$

$ABA_{\mathcal{P}} = \langle \mathcal{L}_{\mathcal{P}}, \mathcal{R}_{\mathcal{P}}, \mathcal{A}_{\mathcal{P}}, \bar{\ } \rangle$:

- **language**: $\mathcal{L}_{\mathcal{P}} = Lit_{\mathcal{P}} \cup NAF_{\mathcal{P}}$
- **rules**: $\mathcal{R}_{\mathcal{P}} = \mathcal{P}$
- **assumptions**: $\mathcal{A}_{\mathcal{P}} = NAF_{\mathcal{P}} = \{not\ a, not\ \neg a, not\ c, not\ \neg c, not\ d, not\ \neg d, not\ e, not\ \neg e\}$
- **contraries**:
  $\overline{not\ a} = a$; $\overline{not\ \neg a} = \neg a$;
  $\overline{not\ c} = c$; $\overline{not\ \neg c} = \neg c$;
  $\overline{not\ d} = d$; $\overline{not\ \neg d} = \neg d$;
  $\overline{not\ e} = e$; $\overline{not\ \neg e} = \neg e$

argument: derivation (modus ponens) from assumptions and rules

$A_{10} : (\{not\ c, not\ d, not\ e\}, \emptyset) \vdash a$

ASP vs ABA

Correspondence between answer sets S and stable extensions $\mathcal{E}$:

- if an argument with conclusion l is in $\mathcal{E}$, then $l \in S$
- if $l \in S$ then an argument with conclusion l is in $\mathcal{E}$
- if for all assumptions *not* l of an argument A, $l \notin S$, then $A \in \mathcal{E}$

$\Rightarrow$ at least one **corresponding argument** for every literal in S
$\Rightarrow$ one with all assumptions "in" S

Attack Trees

For $l \in / \notin S$ and $\mathcal{E}$ the corresponding stable extension:

- start with a (corresponding) argument A with conclusion l
  - A^+ if $A \in \mathcal{E}$
  - A^- if $A \notin \mathcal{E}$
- for any A^+: all attacking arguments are child nodes
  $\Rightarrow$ labelled $-$
- for any A^-: exactly one attacking argument $\in \mathcal{E}$ is a child node
  $\Rightarrow$ labelled $+$

$\Rightarrow$ an argument can have various Attack Trees!

Labelled ABA-Based Answer Set (LABAS) Justifications

For $l \in S$, $\mathcal{E}$ the corresponding stable extension, A a corresponding argument of l, Υ an Attack Tree of A :

- start with l^+
- add for every $+$ argument node in Υ:
  - **support relations** between all assumptions/facts and the conclusion
    $\Rightarrow$ literals and relation $+$
  - **attack relations** between the conclusion of child nodes and the attacked assumption
    $\Rightarrow$ assumption $+$, conclusion and relation $-$
- add for every $-$ argument node in Υ:
  - **support relations** between attacked assumptions and conclusion
    $\Rightarrow$ literals and relation $-$
  - **attack relations** between conclusion of child nodes and the attacked assumption
    $\Rightarrow$ assumption $-$, conclusion and relation $+$

Labelled ABA-Based Answer Set (LABAS) Justifications

For $l \notin S$, $\mathcal{E}$ the corresponding stable extension, $A_1, \ldots, A_n$ all arguments with conclusion l, $\Upsilon_{11}, \ldots, \Upsilon_{1m_1}, \ldots, \Upsilon_{nm_n}$ all Attack Trees of A_i :

- start with l^-
- construct the LABAS Justifications for all Attack Trees as in the positive case

ABA-Based Answer Set Justifications - some properties

explanation in terms of **admissible fragment** of the stable extension $\mathcal{E}$ / the answer set S

Attack Tree for an argument in $\mathcal{E}$:

- set of all A^+ is an **admissible extension**
  $\Rightarrow$ subset of $\mathcal{E}$
- set of all assumptions in all A^+ is an **admissible scenario**
  $\Rightarrow$ subset of NAF literals "in" S

LABAS Justification for a literal in S

- set of all NAF labelled $+$ is an **admissible scenario**
  $\Rightarrow$ subset of NAF literals "in" S

ABA-Based Answer Set Justifications - some properties

explanation in terms of **admissible fragment** of the stable extension $\mathcal{E}$ / the answer set S

Attack Tree for an argument not in $\mathcal{E}$:

- set of all A^+ is an **admissible extension**
  $\Rightarrow$ subset of $\mathcal{E}$
- set of all assumptions in all A^+ is an **admissible scenario**
  $\Rightarrow$ subset of NAF literals "in" S

$\Rightarrow$ admissible fragment attacks argument in question

LABAS Justification for a literal not in S

- set of all NAF labelled $+$ in one of the explanations is an **admissible scenario**
  $\Rightarrow$ subset of NAF literals "in" S

$\Rightarrow$ admissible fragment attacks literal in question