

On the complexity of Temporal Equilibrium Logic

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Temporal Equilibrium Logic (TEL)

[Cabalar and Vega 2007]

- ▶ Answering Set Programming (ASP) capabilities + temporal features of standard LTL.
- ▶ For temporal reasoning not representable in ASP.
- ▶ Temporal extension of propositional **Equilibrium Logic** [Pearce 1996], the latter
 - ▶ well-known logical foundation of ASP;
 - ▶ generalizes stables models of ASP for arbitrary propositional theories.
- ▶ **Non-monotonic semantics**: selection among the models of the monotonic **Temporal logic of Here-and-There** (THT).

THT = LTL + intuitionistic logic of Here-and-There (HT)

Temporal logic of Here and There (THT)

$\varphi ::= \perp \mid p \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid \varphi \rightarrow \varphi \mid \mathbf{X}\varphi \mid \varphi \mathbf{U} \varphi \mid \varphi \mathbf{R} \varphi \quad p \in P$

Temporal logic of Here and There (THT)

$$\varphi ::= \perp \mid p \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid \varphi \rightarrow \varphi \mid X\varphi \mid \varphi U \varphi \mid \varphi R \varphi \quad p \in P$$

Derived modalities:

$\neg\varphi := \varphi \rightarrow \perp$ (negation expressed in terms of implication)

$\top := \neg\perp$

$F\varphi := \top U \varphi$ (eventually)

$G\varphi := \perp R \varphi$ (always)

THT semantics

LTL interpretation: infinite word over 2^P

THT interpretation: (H, T) such that $H \sqsubseteq T$

'There' LTL-interpretation

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$H \sqsubseteq T$ means $H(i) \subseteq T(i)$ for all $i \geq 0$

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$H \sqsubseteq T$ means $H(i) \subseteq T(i)$ for all $i \geq 0$

(H, T) is **total** if $H = T$

THT semantics

$$M = (H, T)$$

$$M, i \not\models \perp$$

$$M, i \models p \quad \Leftrightarrow p \in H(i)$$

$$M, i \models \varphi \vee \psi \quad \Leftrightarrow \text{either } M, i \models \varphi \text{ or } M, i \models \psi$$

$$M, i \models \varphi \wedge \psi \quad \Leftrightarrow M, i \models \varphi \text{ and } M, i \models \psi$$

$$M, i \models \varphi \rightarrow \psi \quad \Leftrightarrow \forall H' \in \{H, T\}, \text{ either } (H', T), i \not\models \varphi \text{ or } (H', T), i \models \psi$$

$$M, i \models X\varphi \quad \Leftrightarrow M, i + 1 \models \varphi$$

$$M, i \models \varphi U \psi \quad \Leftrightarrow \exists j \geq i, M, j \models \psi \text{ and } \forall i \leq k < j, M, k \models \varphi$$

$$M, i \models \varphi R \psi \quad \Leftrightarrow \forall j \geq i, \text{ either } M, j \models \psi \text{ or } \exists i \leq k < j, M, k \models \varphi$$

M is a THT model of φ if $M, 0 \models \varphi$

THT basic properties

$$(H, T), i \not\models \varphi \quad \not\Rightarrow \quad (H, T), i \models \neg\varphi$$

$$(H, T), i \models \varphi \quad \Rightarrow \quad (T, T), i \models \varphi$$

$$(T, T) \models \varphi \quad \Leftrightarrow \quad T \models_{\text{LTL}} \varphi$$

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- Dual temporal modalities independent one from the other one

$$(H, T), i \models F\varphi \quad \not\Rightarrow \quad (H, T), i \models \neg G\neg\varphi$$

$$(H, T), i \models \psi U \varphi \quad \not\Rightarrow \quad (H, T), i \models \neg(\neg\psi R \neg\varphi)$$

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- ▶ THT satisfiability is PSPACE-complete [Cabalar and Demri 2011] (the same complexity as LTL satisfiability [Sistla and Clarke 1985]).

Temporal Equilibrium Logic (TEL)

Non-monotonic semantics: restriction of THT to a subclass of models

A TEL model of φ is a total THT model (T, T) of φ
such that
 $H \sqsubset T$ implies $(H, T) \not\models \varphi$

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- ▶ TEL models: temporal generalization of stable models in propositional ASP.

Negation interpreted as default negation in logic programs.

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$$G(\neg p \rightarrow Xp)$$

Time 0, $\neg p \rightarrow Xp$: p false by default, Xp holds.

Time 1, p and $\neg p \rightarrow Xp$: p true.

Time 2, $\neg p \rightarrow Xp$: ...

The unique TEL model is (T, T) where $T = \emptyset, \{p\}, \emptyset, \{p\}, \dots$

Non-existence of TEL models

LTL satisfiability $\not\Rightarrow$ TEL satisfiability

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- ▶ **Use of nested implication:**
(necessary for non-existence of stable models in Equilibrium Logic)

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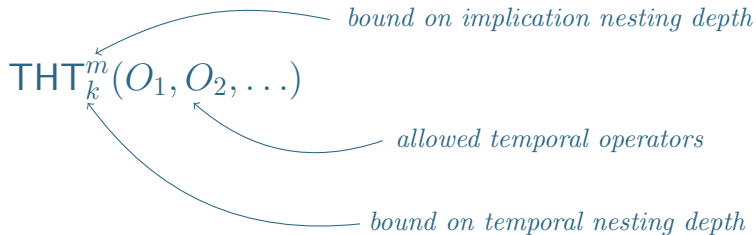
- ▶ **No finite justification for minimal knowledge:**

$$GF p$$

LTL/THT satisfiable but no TEL model.

Investigated problems

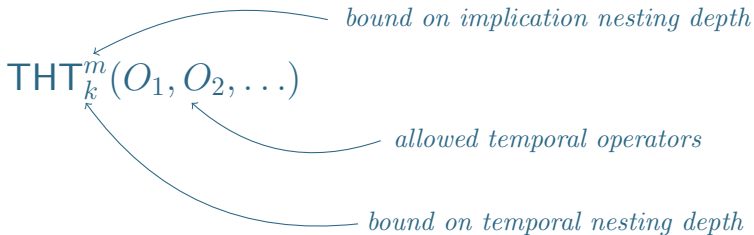
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 - ▶ Systematic analysis of natural THT fragments:



Investigated problems

- ▶ **Complexity of TEL satisfiability.**

- ▶ Systematic analysis of natural THT fragments:



- ▶ **Complexity of minimal LTL satisfiability.**

An LTL model T of φ is *minimal* if $H \not\models_{\text{LTL}} \varphi$ for all $H \sqsubset T$.

EXPSPACE-completeness for TEL satisfiability

TEL satisfiability is known to be in EXPSPACE [Cabalar and Demri 2011].

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Theorem (EXPSPACE lower bounds)

TEL satisfiability is EXPSPACE-complete even for the fragments

$$THT_k^1(F, G, \dots)$$

$$THT_k^m(G, \dots)$$

$$THT_k^m(U, \dots)$$

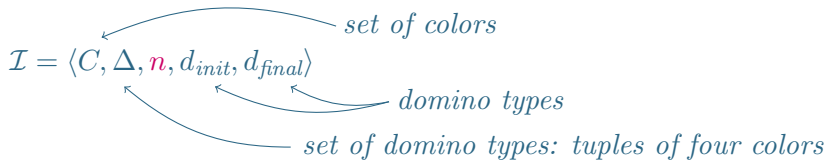
$m \geq 2$ (implication nesting depth) and $k \geq 2$ (temporal nesting depth)

EXPSPACE-hardness for $THT_2^1(F, G)$ is surprising because

- ▶ LTL/THT satisfiability of $THT(F, G)$ is NP-complete [Sistla and Clarke 1985, Cabalar and Demri 2011]
- ▶ Checking equilibrium models for HT^1 formulas is NP-complete.

EXPSPACE-hardness for $\text{THT}_2^1(\mathbf{F}, \mathbf{G})$ (no nesting of implication)

Polynomial-time reduction from a domino tiling problem for grids with exponential number of columns.

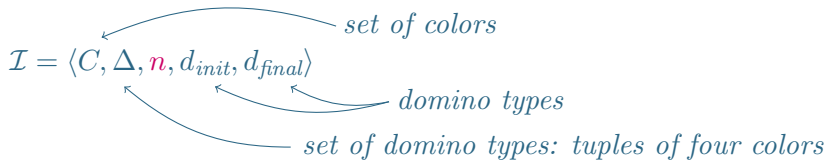


Tilings of \mathcal{I} : grids with 2^n columns and k rows (for some k) such that

- ▶ Each cell contains a domino type;
- ▶ the first cell contains d_{init} ;
- ▶ the last cell is the unique one containing d_{final} ;
- ▶ adjacent cells have the same color on the shared edge.

EXPSPACE-hardness for $\text{THT}_{\frac{1}{2}}(\mathbf{F}, \mathbf{G})$ (no nesting of implication)

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We construct $\varphi_{\mathcal{I}} \in \text{THT}_{\frac{1}{2}}(\mathbf{F}, \mathbf{G})$ such that

$\varphi_{\mathcal{I}}$ is TEL satisfiable \Leftrightarrow there is a tiling of \mathcal{I}

EXSPACE-hardness for $\text{THT}_2^1(\mathbf{F}, \mathbf{G})$ (no nesting of implication)

Encoding of tilings of \mathcal{I} :

$$P_{MAIN} = \Delta \cup [1, n] \times \{0, 1\} \cup \{\$\}$$

- ▶ Cells with content $d \in \Delta$ and column number $i \in [0, 2^n - 1]$ encoded by finite words in

$$\{d\}^+ \{(1, b_1)\}^+ \dots \{(n, b_n)\}^+$$

b_1, \dots, b_n is the binary encoding of column number i .

- ▶ Tilings encoded by finite words over P_{MAIN} listing the encodings of rows from left to right, separated by occurrences of \$.

EXPSpace-hardness for $\text{THT}_2^1(\mathbf{F}, \mathbf{G})$ (no nesting of implication)

We construct $\varphi_{\mathcal{I}}$ over $P = P_{\text{MAIN}} \cup P_{\text{TAG}} \cup \{u\}$

$$\varphi_{\mathcal{I}} = \varphi_{\text{PTC}} \wedge (u \vee \varphi_{\text{bad}})$$

φ_{PTC} captures the pseudo-tiling codes (PTC) (\mathbf{H}, \mathbf{T}) :

- ▶ \mathbf{T} and \mathbf{H} agree on P_{MAIN} and for all i , $\mathbf{T}(i) \cap P_{\text{MAIN}}$ is a singleton;
- ▶ either $\mathbf{T}(i) \supseteq P_{\text{TAG}} \cup \{u\}$ for all i ((\mathbf{H}, \mathbf{T}) is good),
or $u \notin \mathbf{T}(0)$ and $\mathbf{T}(i) \cap P_{\text{TAG}}$ is a singleton;
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- ▶ **Unboundness:** for infinitely many i , $u \in \mathbf{H}(i)$.

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(\mathbf{T}, \mathbf{T}) is non-good: there is non-total PTC (\mathbf{H}, \mathbf{T}) s.t. \mathbf{H} and \mathbf{T} agree on $P \setminus \{u\}$.

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Since φ_{bad} is over $P \setminus \{u\}$.

Remark: every TEL model of $\varphi_{\mathcal{I}}$ is a good PTC.

EXSPACE-hardness for $\text{THT}_2^1(\mathbf{F}, \mathbf{G})$ (no nesting of implication)

$$\varphi_{\mathcal{I}} = \varphi_{PTC} \wedge (u \vee \varphi_{bad})$$

- ▶ for a good total PTC (\mathbb{T}, \mathbb{T}) , no prefix of \mathbb{T} encodes a tiling \Leftrightarrow there is non-total PTC (\mathbb{H}, \mathbb{T}) satisfying φ_{bad} .

EXPSpace-hardness for $\text{THT}_2^1(\mathbf{F}, \mathbf{G})$ (no nesting of implication)

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 - ▶ tag propositions mark local portions of \mathbf{H} : for checking that a bad condition is satisfied.
 - ▶ goodness of (\mathbf{T}, \mathbf{T}) is crucial for ensuring the for each bad condition B in \mathbf{T} , there is a non-total PTC (\mathbf{H}, \mathbf{T}) witnessing B .

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Lemma

The TEL models of $\varphi_{\mathcal{I}}$ are the total good PTC (\mathbf{T}, \mathbf{T}) such that some prefix of \mathbf{T} encodes a tiling of \mathcal{I} .

$\varphi_{\mathcal{I}}$ is TEL satisfiable \Leftrightarrow there is a tiling of \mathcal{I}

Remaining main THT fragments

- ▶ Using only temporal modalities in $\{X, F\}$.
- ▶ No nesting of temporal modalities.
- ▶ No nesting of implication.

Using only temporal modalities X and F

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(T, T) is **almost-empty** if (*) for some i and for all $k \geq i$, $T(k) = \emptyset$.
The *size* of (T, T) is the smallest i satisfying (*).

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Theorem (Small size model property for $THT(X, F)$)

$\varphi \in THT(X, F)$, φ is TEL satisfiable $\Rightarrow \varphi$ has an almost-empty TEL model of size at most $|\varphi|^3$.

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TEL satisfiability of $THT(X, F)$ is Σ_2 -complete. 😊

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- ▶ LTL/THT satisfiability of $THT(X, F)$ is already PSPACE-complete [Sistla et Clarke 1985, Cabalar et Demri 2011]

For $THT(X, F)$, LTL/THT satisfiability is harder than TEL satisfiability!

No nesting of temporal modalities

- ▶ LTL/THT satisfiability of THT_1 is NP-complete
[Demri et al. 2002, Cabalar et al. 2007]
- ▶ TEL satisfiability of THT_1 is NEXPTIME-complete

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- ▶ TEL satisfiability of THT_1 is NEXPTIME-complete
 - ▶ Untractable fragments of THT_1 : ☹️

$$\left. \begin{array}{l} \text{THT}_1^m(\text{F}, \text{G}, \dots) \\ \text{THT}_1^m(\text{U}, \dots) \\ \text{THT}_1^m(\text{R}, \dots) \end{array} \right\} \text{NEXPTIME-complete}$$

$m \geq 2$ (implication nesting depth)

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- ▶ Untractable fragments of THT^1 : ☹️

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THT_1^1

NP-complete

$\text{THT}^1(\mathbf{X}, \mathbf{R})$

$\text{THT}^1(\mathbf{X}, \mathbf{U})$

} PSPACE-complete

No nesting of implication: the fragment $\text{THT}^1(\mathbf{X}, \mathbf{R})$

Remark:

$$\left. \begin{array}{l} \varphi \in \text{THT}^1, \\ \mathbb{T} \text{ minimal LTL model of } \varphi \end{array} \right\} \Rightarrow (\mathbb{T}, \mathbb{T}) \text{ is a TEL model of } \varphi$$

No nesting of implication: the fragment $\text{THT}^1(\mathcal{X}, \mathcal{R})$

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Lemma (Main result for $\text{THT}^1(\mathcal{X}, \mathcal{R})$)

An LTL satisfiable $\text{THT}^1(\mathcal{X}, \mathcal{R})$ formula has a minimal LTL model.

No nesting of implication: the fragment $\text{THT}^1(\mathcal{X}, \mathcal{R})$

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Corollary

For $\text{THT}^1(\mathcal{X}, \mathcal{R})$, LTL satisfiability = TEL satisfiability.

No nesting of implication: the fragment $\text{THT}^1(\mathbf{X}, \mathbf{U})$

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Lemma (Properties of $\text{THT}(\mathcal{X}, \mathcal{U})$)

*Every TEL model of a $\text{THT}(\mathcal{X}, \mathcal{U})$ formula is **almost-empty**.*

No nesting of implication: the fragment $\text{THT}^1(X, U)$

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Lemma (Properties of $\text{THT}(X, U)$)

Every TEL model of a $\text{THT}(X, U)$ formula is *almost-empty*.

Corollary (Main result for $\text{THT}^1(X, U)$)

Let $\varphi \in \text{THT}^1(X, U)$ and $\psi = \varphi \wedge FG \bigwedge_{p \in P(\varphi)} \neg p$.

φ is TEL satisfiable $\Leftrightarrow \psi$ is LTL satisfiable

Proof: \Rightarrow) by the lemma above.

\Leftarrow) if ψ is LTL satisfiable, then φ has a minimal LTL model. By the remark above, φ has a TEL model.

No use of implication: the fragment THT^0

Remark: every THT^0 formula is LTL and THT satisfiable.

Theorem (Lower bound for THT^0)

TEL satisfiability of THT^0 is PSPACE-hard.

Open question: the exact complexity of TEL satisfiability for THT^0 .

Minimal LTL satisfiability

Theorem

Minimal LTL satisfiability is EXPSPACE-complete.

Proof: Lower bound: the same reduction for the lower bound of TEL satisfiability of $\text{THT}_2^1(\text{F}, \text{G})$.

Upper bound: generalization of automata-theoretic approach for LTL satisfiability.

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Proof: Lower bound: the same reduction for the lower bound of TEL satisfiability of $\text{THT}_2^1(\text{F}, \text{G})$.

Upper bound: generalization of automata-theoretic approach for LTL satisfiability.

- ▶ Minimal LTL satisfiability versus TEL satisfiability: different costs for THT fragments.
 - ▶ **Example:** for THT_1 , minimal LTL satisfiability is NP-complete, while TEL satisfiability is NEXPTIME-complete.

Discussion: wrap up

- ▶ Systematic analysis of complexity of TEL satisfiability for natural THT fragments.
 - ▶ No difference between implication (resp., temporal) nesting depth 2 and $k > 2$.
 - ▶ $\text{THT}(X, F)$: the unique tractable fragment with both nesting of implication and nesting of temporal modalities.
 - ▶ Different computational cost of dual temporal modalities.
Example: for $\text{THT}(G)$, EXPSPACE-completeness; for $\text{THT}(X, F)$, Σ_2 -completeness.

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Example: for $\text{THT}(G)$, EXPSPACE-completeness; for $\text{THT}(X, F)$, Σ_2 -completeness.
- ▶ Complexity of minimal LTL satisfiability.
 - ▶ **LTL over *finite* words**: LTL satisfiability = minimal LTL satisfiability.
 - ▶ **LTL over *infinite* words**: minimal LTL satisfiability exponentially harder than LTL satisfiability.

Discussion: perspectives

- ▶ Expressiveness issues for TEL fragments:
 - ▶ Kind of temporal problems expressible in tractable fragments.
 - ▶ Is the syntactical hierarchy of considered THT fragments semantically strict w.r.t. THT or TEL semantics?

Known results: the hierarchy of $\text{THT}_m(\mathbf{U})$ fragments is strict w.r.t. LTL semantics [Etesami et Wilke 1996].

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- ▶ Characterization of TEL languages:
 - ▶ Known results: TEL languages are ω -regular [Cabalar et Demri 2011].
 - ▶ Conjecture: TEL languages are LTL definable!

MANY THANKS ☺