In Defense of Logical Minds

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Abstract

According to the received view in the psychology of reasoning, Piaget's view that

$\mathcal{F}$ Humans naturally develop a context-free deductive reasoning scheme at the level of elementary first-order logic.

has been overthrown by the poor performance of educated adult subjects on specific logic problems (e.g., Wason's selection task). We propose that Piaget's $\mathcal{F}$ (or at least a variant) is alive and well, because the subjects in question are simply victims of a defective education. With a modicum of the right sort of logic training, humans reason deductively on logic problems well enough to vindicate Piaget.

The Received View: Piaget Dead on Deductive Reasoning

While some elements of Piaget's thought remain very much alive today, the consensus seems to be that at least one part has long been reduced to a carcass: the part according to which

$\mathcal{F}$ Humans naturally develop a context-free deductive reasoning scheme at the level of elementary first-order logic.[1]

As evidence that $\mathcal{F}$ is generally regarded to be stone cold dead, one can do no better than Peter Wason's [Wason, 1995] relaxed remarks in his contribution to a recently published book [Newstead and Evans, 1995] written in his honor. Wason is credited with devising the seminal experiments that led to the rejection of $\mathcal{F}$ (we will visit two of the experiments below), and the remarks in question arise from his retrospection on these experiments. For example, we read: "The first formal experiments, done partly in Scotland, met with grave looks from dedicated Piagetians; the subjects' were clearly incompatible with 'formal operations' " ([Wason, 1995], 296). Wason writes here and elsewhere as if $\mathcal{F}$ has been long buried; most others in the psychology of reasoning follow suit. For example, the other contributors to the volume in question, each and every one of them, is like-minded: they either explicitly reject or presuppose the falsity of Piaget's $\mathcal{F}$.[2]

In this short paper we present a prolegomenon for vindicating Piaget's affirmation of $\mathcal{F}$ and its relatives. The paper is structured as follows. In section 2 we present four logic problems on which the vast majority of educated adult subjects do indeed exhibit poor performance: Wason's selection task and "THOG" problem, a well-known and thoroughly studied invalid syllogism [Oakhill et al., 1989], and an ingenious fourth problem involving exclusive disjunction recently introduced by Johnson-Laird and Savary [Johnson-Laird and Savary, 1995].[3] In section 3 we briefly review the main responses that have developed in response to the experiments and data discussed in section 2. In section 4 we briefly present our response. In the final section, 5, we discuss empirical support for our response, garnered from S. Bringsjord's attempt to produce in students the reasoning ability Piaget believed would develop at the stage of "formal operations." This discussion is accompanied by synoptic formal analysis of two of the problems presented in section 2.

Four Puzzles

The psychology of reasoning is in many ways driven by a smallish set of classic reasoning problems. First and foremost among these is no doubt the Wason selection task.[4]. If you haven't seen this before, go ahead and try to solve it, and record your answer and justification.

[1]That is, at the level of the propositional calculus plus command over some set of simple operations involving the quantifiers 'some' ($\exists$ in FOL) and 'all' ($\forall$ in FOL)). The proposition $\mathcal{F}$, or at least a thesis very close to it (more about variants on $\mathcal{F}$ below), is articulated and defended by Piaget and Inhelder in [Inhelder and Piaget, 1958].

[2]For example, as Johnson-Laird (sarcastically?) says:

> It seems that adult subjects in the selection task have not reached the Piagetian level of formal operations. Yet they are supposed to have attained it around the age of 12. ([Johnson-Laird, 1995], 133)

[3]As cognoscenti know, Johnson-Laird worked with Wason in the "early days" to devise the experiments taken by nearly all to overthrow the likes of $\mathcal{F}$ (e.g., see [Wason and Johnson-Laird, 1972]).

[4]Wason first described this problem in print in [Wason, 1966].

P1: Wason's Selection Task

Suppose that I have a pack of cards each of which has a letter written on one side and a number written on the other side. Suppose in addition that I claim the following rule is true:

- If a card has a vowel on one side, then it has an even number on the other side.

Imagine that I now show you four cards from the pack:

$\boxed{\text{E}}$ $\boxed{\text{T}}$ $\boxed{4}$ $\boxed{7}$

Which card or cards should you turn over in order to decide whether the rule is true or false?

- **Answer**:
- **Justification**:

Only about 5% of the educated population give the correct answer, which is E and 7. If you said (only) E, you saw that the rule in question would be overthrown were there to be an odd number on the other side of this card — but you failed to note that if the 7 card has a vowel on the other side, this too is a case that shoots down the rule.

P2: The THOG Problem

Here's a second problem, once again from Peter Wason; try again to solve it, and record your answer and justification.[5]

Suppose that there are four possible kinds of objects:

- an unhappy dodecahedron
- a happy dodecahedron
- an unhappy cube
- a happy cube

Suppose as well that I have written down on a hidden piece of paper one of the attitudes (unhappy or happy) and one of the shapes (dodecahedron or cube). Now read the following rule carefully:

- An object is a GOKE if and only if it has either the attitude I have written down, or the shape I have written down, but not both.

I will tell you that the unhappy dodecahedron is a GOKE. Which of the other objects, if any, is a GOKE?

If your answer is 'happy cube' you are right. If you missed this problem (known originally as the THOG problem) you're not alone: only about 10% of the educated adult population gets the right answer.[6]

[5]This is Wason's [Wason, 1977] THOG problem, with the shape-names changed to work alongside a representation of this problem in the HYPERPROOF system [Barwise and Etchemendy, 1994] S. Bringsjord uses to teach logic, automated theorem-proving, and AI. HYPERPROOF is discussed below.

[6]Actually, in 12 years of presenting the selection task and the THOG problem, S. Bringsjord has never obtained only a 5% differential between it and P1. In his experience at Brown University and Rensselaer, students find the THOG problem *much easier* than the selection task. E.g, in *General Psychology* of Fall 1997, 10% of the students (at the start of the semester) solved the selection task, but over 30% solved the THOG. In a recent offering of (S. Bringsjord's) *Introduction to Logic Programming*, 80% solved THOG, 60% the selection task.

Though these problems are still catalyzing new research today, they were devised decades ago by Wason. Let's turn now to a recent problem, an ingenious one devised by Johnson-Laird [Johnson-Laird and Savary, 1995]:[7]

P3

What can you infer about the hand in question from the following if-then statement?

- If there is a king in the hand, then there is an ace, or else if there isn't a king in the hand, then there is an ace.

Many subjects infer that there must be an ace in the hand. But alas, this is wrong. (As to why, if you don't know, we're afraid we're going to keep you hanging till we formally diagnose this problem in the final section).

Let's round things out with a fourth logic problem presented in [Oakhill et al., 1989]:

P4

What follows from the following two premises?

(1) All the Frenchmen in the room are wine-drinkers.
(2) Some of the wine-drinkers in the room are gourmets.

Many subjects in this case infer

(3) Some of the Frenchman in the room are gourmets.

But (3) doesn't follow from (1) and (2). (Do you see precisely why? Once again, formal diagnosis is given later on.)

Responses

There are three main responses to the fact that people are dreadful at solving the quartet presented in the last section:[8]

The Mental Logic Response According to this response, which is promoted (e.g.) by Lance Rips [Rips, 1994] and David O'Brien [O'Brien, 1995], humans do naturally acquire the ability to deduce abstractly, but they are restricted to a deductive scheme having considerably less power than standard first-order logic — and this scheme is not adequate to crack problems like those seen in the quartet displayed in the previous section.

[7]One of us (Noel) notes that this particular problem may be confusing to people with some background in computer programming. (Do you see why?) We are planning an experiment running in parallel to J-L's in which the phrase 'or else if' is replaced with 'or' and the phrase ', but not both' is added at the end.

[8]As we say, these are the *main* responses. One response we leave aside holds that there is a procedure for "checking for cheaters" that has evolved in us as a result of natural selection (e.g. see [Cosmides, 1989]). This response appeals to much of the same experimental data appealed to by proponents of the Pragmatic Reasoning Schemas Response.

The Pragmatic Reasoning Schemas Response This response — the chief advocates for which are Cheng and Holyoack (e.g., [Cheng and Holyoak, 1985], [Holyoak and Cheng, 1995]) — springs from the fact that when conditionals like the one seen in the selection task are changed into "deontic" conditionals, performance improves substantially. For example, Griggs and Cox [Griggs and Cox, 1982] showed that deontic conditionals like

- If a person is drinking beer then the person must be over 18.

tended to elicit correct selections. Here is an example of a schema that the previous conditional might be an instantiation of:

- If the precondition is not satisfied, then the action must not be taken.

The Mental Models Response The response has been championed for quite a while now by Johnson-Laird (e.g., [Johnson-Laird, 1995], [Johnson-Laird, 1997]). Logicians will[9] identify the response with semantic tableaux, but Johnson-Laird prefers a somewhat idiosyncratic specification, and he has recently produced a computer program that instantiates this specification. According to this program, a disjunction $\phi \vee \psi$ is represented by two models, one in which ϕ obtains, and one in which ψ obtains; the two models are written (one to a line) as

$$\phi$$

$$\psi.$$

If the reasoner now learns that ϕ is false, she strikes out the first model and is left with the second — in which ψ is true. (This then becomes a mental models version of unit resolution.)[10]

Now as a matter of fact we see fatal problems infecting each of these responses (as well as the others we don't explicitly consider),[11] but our objective here is just to air the responses in order to place our own in the full context of the psychology of reasoning and Piaget's $\mathcal{F}$. We turn now to our own response.

Our Response

Put brutally, our response is this. The reason that subjects perform poorly on problems like those seen in the quartet above is that their education is defective; and because their education is defective, they haven't reached Piaget's stage of formal operations. We realize, of course, that $\mathcal{F}$ uses the term 'naturally,' and we realize as well that this connotes that people, *without special training*, will reach the competence in question.[12] But this is a bluff we're quite willing to call. What, precisely, does 'naturally' mean? We all know that without special training humans aren't able to solve even simple arithmetic problems. For example, consider this problem:

- John is given $\frac{3}{4}$ of a chocolate chip cookie. Each of his ten friends will be content if they receive $\frac{1}{8}$ of such a cookie. If John is willing to keep none for himself, and he can divide his cookie-part precisely, how many friends can he satisfy?

Even educated adults do poorly on this problem.[13] (At a recent talk at a major university, S. Bringsjord found, upon presenting this problem, that a goodly number of *professors* had completely forgotten how to divide fractions.) Does poor performance of many subjects on a puzzle like this imply the falsity of some such proposition as the following one?

$\mathcal{F}_A$ Humans naturally develop a context-free scheme at the level of elementary arithmetic.

If not, then why should $\mathcal{F}$ fall? Perhaps, again, the problem pertains not to underlying cognitive development, but rather to education, pure and simple. If someone insists on a rather strict reading of 'naturally,' according to which only a bare minimum of "official" education is required to support the ascription of the adverb naturally, and therefore according to which $\mathcal{F}_A$ is indeed taken to be false, then we will be quite content to settle for defending the view that

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[9] Correctly, in S. Bringsjord's opinion: he thinks that mental models is provably reducible to semantic tableaux.

[10] What we have just said is in no way offered as a thorough description of the mental models approach.

[11] Here is the tip of the iceberg of problems we see:

- The permission schema reaction fails to account (e.g.) for consistently better-than-guessing performance on syllogisms. Subjects can be given a choice between four conclusion of the A, E, I or O form, as well as "no conclusion follows," producing a chance rate of 20% for correct solution. A correct response rate of more than 50% is common; one of us (S. Bringsjord) has replicated this rate many times in my psychology, logic, and computer science classes.

- Pragmatic reasoning schemas by definition fail to apply to (or to explain competence with) abstract reasoning problems having nothing to do with pragmatic reasoning.

- Both the mental logic and mental models approach fails to "scale up" to logic problems more difficult than the simple ones that dominate the psychology of reasoning literature.

[12] We realize also that part of what we've been calling the 'received view' is that some such term is part of the thesis at stake. For example, in their discussion of the psychology of reasoning, Stillings et al. [Stillings et al., 1995] opine that a proposition virtually identical to $\mathcal{F}$ is "obviously" overthrown by the fact that the vast majority of subjects fail to solve problems like those visited in our quartet.

[13] Here's another problem with a slightly different twist:

- 293 students from Grover Middle School are going on a field trip to New York City. Each bus carries 32 students. How many buses will be needed for the trip?

$\mathcal{F}'$ If educated in logic as they are in arithmetic, humans develop a context-free deductive reasoning scheme at the level of elementary first-order logic — a scheme that will allow for the solving of problems like those seen in our quartet above, and significantly harder problems as well.

We see at least two general ways to reply to our view; in a nutshell they are:

1. "You are stretching the concept of 'naturally acquire' too far. $\mathcal{F}'$ is true, but this doesn't vindicate Piaget."
2. "$\mathcal{F}'$ is false — or at least you've done nothing to convince us that it's true."

The first complaint seems to be easy enough to handle. Clearly, $\mathcal{F}'$ is firmly in the spirit of Piaget, and we would be quite content with having defended him to this degree. Besides, the point of the reference in $\mathcal{F}'$ to arithmetic is to limit the training in logic to something well short of sustained and intense training of the sort an aspiring mathematician or logician would encounter. The training in question is supposed to be analogous to what people receive in arithmetic in the normal course of development in civilized society.[14] The second objection is more formidable. In fact, some readers will be of the opinion that this objection is *very* formidable — because apparently training in logic *doesn't* cause facilitation on problems like those seen in the above quartet (see [Cheng et al., 1986]). One of us (S. Bringsjord) confesses that he has long found the claim that logic training fails to facilitate on problems like our quartet nothing short of astonishing. After all, all four of the problems above (and, indeed, *all* logic problems at the heart of the psychology of reasoning), from the standpoint of the content of a first-course in mathematical logic, are painfully simple. What aspect of the training could be preventing facilitation?

Empirical Support for Our Response

One of us (S. Bringsjord) hypothesized that the training provided, because it is confined to merely brisk coverage of certain purely syntactic rules of inference, fails to give students/subjects the formal tools required for solving (P1)–(P4), and other problems from the same general class.[15] What might be missing, S.B. specifically hypothesized, is (among other things)

H1 teaching of *dis*proofs;

H2 teaching of diagrammatic techniques;

H3 and teaching of rigorous and general-purpose procedures for formalizing natural language logic problems in first-order logic so that they can then be solved by proof.

In order to test these hypotheses, S.B. has repeatedly taught students how to use certain theorem-proving technologies in order to produce proofs that constitute solutions to problems at the level of our quartet, and well beyond. These technologies allow for both the construction of disproofs (so-called "proofs of non-consequence") and for diagrammatic reasoning; and the use of these systems to generate proofs absolutely requires that students be able to formalize natural language logic problems in first-order logic so that such problems can be mechanically solved by proof.

In three separate pre-test/post-test designs, using the system HYPERPROOF [Barwise and Etchemendy, 1994] (and to some degree the system OTTER; both are treated by S. Bringsjord in class as theorem-provers, though the former is designed mostly for pedagogical purposes while the latter is used by professional mathematicians), this is precisely what was found. For example, in the first experiment, while 20% of students at the start of a HYPERPROOF-based first course in mathematical logic solved the selection task, P1, over 80% solved a formal analogue at the end of the course. Similarly dramatic improvement was obtained for P2-P4. More importantly, at the conclusion of the course students were able to routinely solve logic problems *much more difficult* than P1–P4. This level of performance was replicated twice in the

[14] This is as good a place as any to say that at least one of us (S. Bringsjord) does happen to believe that careful reading of Piaget's work on the issues before us — e.g., [Inhelder and Piaget, 1958] and [Beth and Piaget, 1966] — reveals that a sucessful defense of $\mathcal{F}'$ constitutes his vindication in the area of logical reasoning, but exegesis of his writings will have to wait for a time when we have more space.

[15] The training in question, unfortunately, is probably equivalent to that required and supplied in a number of states as part of their K-12 math curricula. For example, students in New York State are taught to manipulate symbols in the propositional calculus, but they are not taught any of the things we enumerate immediately below in my hypotheses H1–H3. To make the point a bit more focussed, students in New York State are taught how to respond to questions like

> Given the statements
>
> $\neg a \lor \neg b$
> b
> $c \rightarrow a$
>
> which one of the following statements must also be true? (Check the correct answer.)
>
> ☐ c
> ☐ $\neg b$
> ☐ $\neg c$
> ☐ h
> ☐ a
> ☐ none of the above

But they are not taught how to disprove the incorrect answers here, nor how to use established diagrammatic techniques to carry out proofs and disproofs at this level, *nor* how to take an English word (logic) problem and transform it into a formal representation that can then be used to mechanically generate a solution in the form of a proof.

two other experiments. Of course, Piaget predicted that deductive operations would be something a young person would master (with suitable interchange with the environment). We see no reason to think that, say, sixth graders couldn't routinely learn to do what my college students routinely learn to do; and we're planning experiments at this grade level to see if we're correct.[16]

Let us conclude by briefly diagnosing the two members of the quartet P1–P4 in the context of my hypotheses and the pedagogy with which they are associated. (More details can be found by looking at the courses on S. Bringsjord's web site.)

Consider first the selection task in connection with H3. Students were told that they had to cast the selection task in a form that would allow the automated theorem prover OTTER to solve it. Here is a sample input file:

```
% Wason's Selection Task
% This is an input file for the
% proof that if card
% 4 (which shows a 7), when flipped,
% reveals a vowel,
% then a contradiction results.

set(auto).

formula_list(usable).
% The rule:
all x (Vowel(x) -> Even(x)).
% What we observe facing us
% before flipping:
Vowel(c1).
Consonant(c2).
Even(c3).
Odd(c4).
% The fact that even and odd
% numbers are distinct:
all x (Even(x) <-> -Odd(x)).
% The possibility that card
% 4 bears a vowel:
Vowel(c4).
end_of_list.
```

And here is the proof from a sample output file (`$F` indicates that a contradiction has been found):

```
---------------- PROOF ----------------
1 [] -Vowel(x)|Even(x).
2 [] -Even(x)| -Odd(x).
6 [] Odd(c4).
8 [] Vowel(c4).
10 [hyper,8,1] Even(c4).
11 [hyper,10,2,6] $F.
------------ end of proof -------------
```

Students get so good at this process that even logic problems much harder than P1 can be formalized and sent to OTTER to be solved by proof. For example, they are soon able to solve a problem like the Dreadsbury Mansion Mystery:[17]

> Someone Who lives in Dreadsbury Mansion killed Aunt Agatha. Agatha, the butler, and Charles live in Dreadsbury Mansion, and are the only people who live therein. A killer always hates his victim, and is never richer than his victim. Charles hates no one that Aunt Agatha hates. Agatha hates everyone except the butler. The butler hates everyone not richer than Aunt Agatha. The butler hates everyone Agatha hates. No one hates everyone. Agatha is not the butler.
>
> Now, given the above clues, there is a bit of a disagreement between three (incompetent?) Norwegian detectives: Inspector Bjorn is sure that Charles didn't do it. Is he right? Inspector Reidar is sure that it was a suicide. Is he right? Inspector Olaf is sure that the butler, despite conventional wisdom, is innocent. Is he right?

Let's consider now the teaching of disproofs, and let's anchor the discussion to P4: We noted above that in P4 (3) doesn't follow from (1) and (2). This sequence — (3) from (1) and (2) — is one of 256 syllogisms studied by Aristotle (out of this space, 15 are valid). It's form is

$$\begin{array}{c} \text{All } As \text{ are } Bs \\ \text{Some } Bs \text{ are } Cs \\ \hline \text{Some } As \text{ are } Cs \end{array}$$

In order to conform to the geometric predicates built into the HYPERPROOF system [Barwise and Etchemendy, 1994], let's recast this inference in first-order logic as follows.

$$\begin{array}{c} \forall x(\mathtt{Dodec}(x) \Rightarrow \mathtt{Happy}(x)) \\ \exists x(\mathtt{Happy}(x) \land \mathtt{Large}(x)) \\ \hline \exists x(\mathtt{Dodec}(x) \land \mathtt{Large}(x)) \end{array}$$

Now, it is possible to prove in Hyperproof that this inference is invalid. The finished proof is shown in Figure 1. You will notice that the first two lines of symbolic text match the premises; this is the "given" information. You will then see what is called a "sub-proof" under these lines; the sub-proof is composed of three lines. In the first line of the sub-proof (an assumption) a visual situation is constructed in which two dodecs (frenchmen) are happy (wine-drinkers), and some happy things (wine-drinkers) are large (gourmets). In the second line of the sub-proof, a check is done to make sure that this visual situation is consistent with the given information (a $\surd$ indicates that we have consistency). Finally, in the last line, we simply observe that the conclusion in question is false in the visual situation. This constitutes a disproof of the syllogism.

In conclusion and in short, Piaget is right: humans can rather easily reach the level of formal deductive operations if they are educated in logic as they are in areas like reading and arithmetic.[18]

[16] A fuller description of the experiments conducted, as well as those we're planning, is forthcoming.

[17] This problem, posed in HYPERPROOF, can be obtained from S. Bringsjord's web site (under *Intro to Logic*). The problem formalized for and solved by OTTER can be found on the site as well (under *Intro to Logic Programming*).

[18] We have no doubt that you have by this time solved P2, the THOG problem. But perhaps you would like the answer to P3? (If not, stop reading this footnote now.) Since the disjunction in P3 is a so-called *exclusive* one, we can represent it by $[(k \Rightarrow a) \lor (\neg k \Rightarrow a)] \land \neg[(k \Rightarrow a) \land (\neg k \Rightarrow a)]$. It is easy to prove that one of the conditionals must be false. (Intuitively, if both can't be true, but one of them is true, this leaves one as false.) But since (as follows from the truth-table for $\Rightarrow$) the only

Figure 1: The "Frenchmen" Syllogism Disproved in Hyperproof

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