SPATIAL COGNITION AND COMPUTATION, *vol*(issue), 1–end

The Space within Fisherman's Folly: playing with a puzzle in mereotopology

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In this paper we propose a spatial ontology for reasoning about holes, rigid objects and a string, taking a classical puzzle as a motivating example. In this ontology the domain is composed of spatial regions whereby a theory about holes is defined over a mereotopological basis. Within this theory we define a data structure, named chain, that facilitates a clear and efficient representation of the puzzle states and its solution.

Qualitative Spatial Reasoning, Commonsense Reasoning

1 Introduction

Understanding the reasoning processes involved in spatial knowledge is one of the key issues in the investigation of cognition, as space not only shapes our actions in the commonsense world, but also serves as the scenario in which our everyday experiences take place. Research in *Qualitative Spatial Reasoning* (QSR) (Stock, 1997; Ligozat et al., 2004) attempts the logical formalisation of spatial knowledge based on primitive relations defined over elementary spatial entities. For instance, QSR theories include a mereotopological theory based on the connectivity between spatial regions (Randell et al., 1992), the definition of occlusion and parallax (Randell et al., 2001; Randell and Witkowski, 2002), spatial vagueness (Cohn et al., 1997; Guesgen, 2002a), the abductive assimilation of sensor data (Santos and Shanahan, 2002; Santos and Shanahan, 2003; Santos, 2007), as well as the definition of qualitative theories about distance (Hernández et al., 1995; Guesgen, 2002b), boundaries (Meathrel and Galton, 2001), shapes (Schlieder, 1996; Clementini and Felice, 1997) and so forth (Cohn and Hazarika, 2001).

This work investigates, from a QSR perspective, the spatial knowledge of a do-

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main composed of non-trivial objects such as a *string* and *holed objects*. To this aim, we take as a starting point the formalisation of puzzle-like examples, since these domains offer a small number of objects while keeping enough complexity for a challenging problem of knowledge representation. We present a first attempt to capture the spatial ontology underlying a spatial puzzle, called the Fisherman's Folly. An automated solution to this puzzle was previously proposed in (Cabalar and Santos, 2006) although that work was strictly tackled from a planning perspective, without really deepening into the underlying spatial features. In the current paper we are concerned instead with the explicit formulation of the spatial relations involving the relative location of objects with respect to holes. We also show how holes entry boundaries can be used to provide a suitable representation of the arrangement of objects in this domain (including the string) allowing an automated solution to the puzzle. In order to accomplish this task, we draw some attention on how actions can be executed in this domain in order to achieve a pre-defined goal.

First of all, let us describe the motivating spatial puzzle assumed in this work.

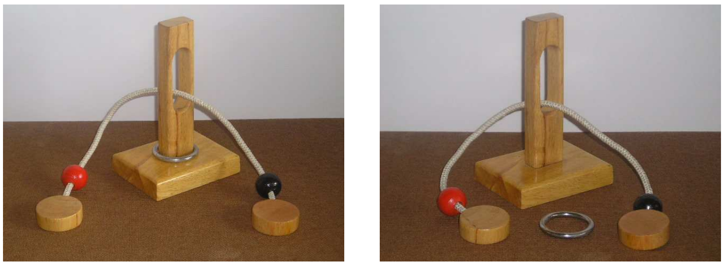
The Fisherman's Folly puzzle

The elements of the puzzle are a holed post (P) fixed to a wooden base (B), a string (Str), a ring (R), a pair of spheres (S_1, S_2) and a pair of disks (D_1, D_2). The spheres can be moved along the string, whereas the disks are fixed at each string endpoint. The string passes through the post's hole in a way that one sphere and one disk remain on each side of the post. It is worth pointing out that the spheres are larger than the post's hole, therefore the string cannot be separated from the post without cutting either the post, or the string, or destroying one of the spheres. The disks and the ring, in contrast, can pass through the post's hole. We should also mention that the disks do not fit into the ring hole, through where the spheres can pass freely if it is not already occupied by another object.

In this work we assume that neither the length nor the thickness of the string constrain any solution to the puzzle, i.e. the string is infinitely extensible and one-dimensional. Relaxing these assumptions is a matter for future work.

In the initial state (shown in Figure 1(a)) the post is in the middle of the ring, which in its turn is supported on the post's base. The goal of this puzzle is to find a sequence of transformations that, while maintaining the physical integrity of the domain objects, allow us to free the ring from the rest of objects, regardless their final configuration. Figure 1(b) shows one possible goal state.

Amongst the domain objects, we have also to consider four holes in order to provide an automated solution to the puzzle. The holes in the puzzle are: the post hole (Ph), the ring hole (Rh) and the two sphere holes (Sh_1 and Sh_2). The domain entities can be classified into three different sorts: *long objects*, *regular objects* and *holes*, corresponding in the puzzle to the sets $\{P, Str\}$, $\{P', R, S_1, S_2,$



(a) Initial (b) Goal: the ring must be free

Figure 1.: A spatial puzzle: the Fisherman's Folly.

$D_1, D_2, B\}$ and $\{Ph, Rh, Sh_1, Sh_2\}$, respectively. For our purposes, we consider that the sort *long objects* represents elliptic cylindrical bodies whose major axis is much larger than their mean and minor axes; whereas *regular objects* identifies the remainder puzzle objects that are not holes. We actually divide the post into its top part P' containing the hole, and its bottom part P, below the hole. In this work we only consider permeability through holes, ruling out of our domain semisolid objects (such as sponges, gelatinous bodies etc).

The goal of this paper is to formalise the Fisherman's Folly puzzle in terms of an ontology about holes built upon mereotopological relations (described in Section 2). We also show how this ontology can be used to define a data structure that provides a clear and efficient representation of the puzzle states (Section 3) and the state changes towards the puzzle solution (described in Section 4).

2 A theory about holes

In this section we follow the guidelines proposed in (Varzi, 1996; Casati and Varzi, 1999) and construct a basic ontology about holes using mereotopological relations. In order to accomplish this task, in this work the puzzle objects are identified with their occupancy regions. We define a *hole* h in an object x as the spatial region constituting that portion of x's complement that lies inside x's occupancy region. Object x receives the name of *host* of hole h.

There are at least three distinct types of holes: *cavities*, i.e. holes that are entirely hidden inside their hosts; *hollows*, which are superficial depressions on the host; and, perforating holes (or tunnels), which are holes that have at least two distinct entry boundaries. As we will exclusively focus on the latter, from now on by a *hole* we will mean a perforating hole.

In the formalisation described below, holes are assumed as open regions whose boundaries belong to their host objects. The relationship between holes and their hosts is formalised using the elementary relation: $H(h, x)$, meaning "h is a hole in the object x" (conversely, "x is the host of h") (Casati and Varzi, 1999). For example, in the puzzle domain we have: $H(Rh,R) \wedge H(Sh_1,S_1) \wedge H(Sh_2,S_2) \wedge H(Ph,P')$. We will include the constraint:

$$H(h,x) \wedge H(h,y) \rightarrow x = y \tag{1}$$

which asserts that a hole has a unique host. Although this restriction was not present in Casati & Varzi's formalisation[1], it will allow simplifying the formulation in the current context.

As we assumed that the space is only populated by spatial regions, apart from the relation $H/2$, it is convenient to include in the basic theory about holes a set of mereotopological relations accounting for the connection and the part-whole relations between spatial regions. In this work we use RCC-8 (Randell et al., 1992) which is a first-order axiomatisation of spatial relations based on a dyadic primitive relation of *connectivity* ($C/2$) between two regions. Informally, assuming two regions x and y, the relation $C(x,y)$, read as *"x is connected with y"*, is true if and only if the *closures* of x and y have *at least a* point in common. Assuming the $C/2$ relation as primitive, and that x, y and z are variables for spatial regions, the following mereotopological relations can be defined: $DC(x,y)$, which stands for "x is disconnected from y"; $EQ(x,y)$, for "x is equal to y"; $PO(x,y)$, for "x partially overlaps y"; $EC(x,y)$, for "x and y are externally connected"; $TPP(x,y)$, for "x is a tangential proper part of y"; $NTPP(x,y)$, for "x is a non-tangential proper part of y"; and, $TPPi/2$ and $NTPPi/2$ are the inverse relations of $TPP/2$ and $NTPP/2$ respectively. These relations are depicted in Figure 2. We write $PP(x,y)$ ("x is a proper part of y") to stand for $TPP(x,y) \vee NTPP(x,y)$.

Here we follow the original interpretation of $C/2$ as proposed in (Randell et al., 1992) (and further explained in (Cohn et al., 1997)). Although there are point-free interpretations of connectivity, understanding $C(x,y)$ holding when *the topological closures of x and y share at least one point* allows us to represent an object position with respect to a hole in an appropriate way, as we shall see further in this paper.

Assuming RCC, the relation $H(h,x)$ can be constrained by Axioms (2) and (3) below. Axiom (2) guarantees that the host of a hole is not itself a hole; whereas Axiom (3) states that the hole and it's host object are externally connected (Casati and Varzi, 1999).

$$H(h,x) \rightarrow \neg H(x,y) \tag{2}$$

$$H(h,x) \rightarrow EC(h,x) \tag{3}$$

Moreover, Axiom 2 implies that the relation H is irreflexive (meaning that no hole hosts itself) and asymmetric (i.e., the host cannot be a hole of its hole). In

[1] As an example, in the general case, for instance, we could have that x is a part of a compound object y, and that both $H(h,x)$ and $H(h,y)$.

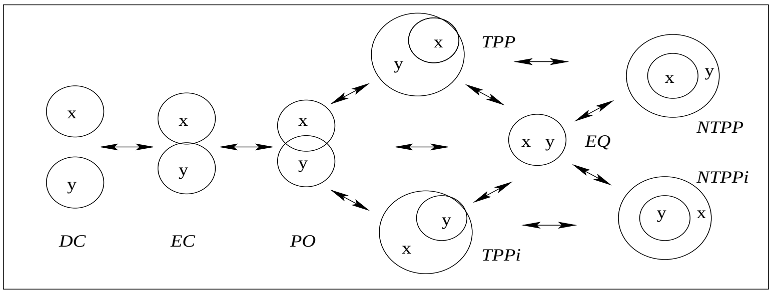


Figure 2.: The RCC8 relations and their conceptual neighbourhood diagram.

this way, we can establish a first classification of regions into two sorts, *holes* and *non-holes*, defining the sort predicates:

$$\begin{aligned} Hole(h) &\equiv \exists y H(h, y) \\ Nonhole(x) &\equiv \neg Hole(x) \end{aligned}$$

In the rest of the paper, we will use the sorted variables h, h' to denote holes and sorted variables x, y to denote non-holes, whereas v, w will be used for regions of any kind.

2.1 Penetrating objects

An essential characteristic of holes is that they can be penetrated by other objects. Therefore, the hole ontology has to include relations about the relative location of a hole with respect to the penetrating object. In a world uniquely populated by spatial regions, relative location can be expressed by mereotopological relations. In order to define relative location with respect to a hole, we need the concept of a hole *entry boundary* (EB) that is defined in (Casati and Varzi, 1999) by the relation $EB(h^i, h, x)$, read as "h^i is a maximally connected part of the hole h (fiat) boundary that is nowhere a boundary of the host x". In our case, as the host x of h is uniquely defined, we can just represent this as $EB(h^i, h)$ assuming that $Hole(h)$ is true. If a hole h has n entry boundaries, we will usually denote them as h^i with $1 \leq i \leq n$ (as we deal with perforating holes, $n \geq 2$). However, when $n = 2$ we will also write h^- and h^+ in place of h^1 and h^2 respectively. This is the case, for instance, of the four holes in the Fisherman's Folly puzzle, so that we would deal with the facts $EB(Ph^-, Ph)$, $EB(Ph^+, Ph)$, $EB(Rh^-, Rh)$, $EB(Rh^+, Rh)$, and $EB(Sh_j^-, Sh_j)$, $EB(Sh_j^+, Sh_j)$ for the two spheres $j = 1, 2$.

This paper assumes that any entry boundary h^i (and any other spatial region in this work) has the same dimension as the space within which it is embedded[2]. This assumption guarantees the intended interpretation of RCC relations in the present work.

We can now express the following relations with respect to an object x and a hole h:

- $WOut(x, h)$, read as "x is *wholly outside* h," if and only if $DC(x, h)$;
- $JOut(x, h, h^i)$, read "x is *just outside* h with respect to entry boundary h^i," and equivalent to

$$\exists y\ H(h, y) \wedge EB(h^i, h) \wedge (EC(x, h^i) \vee PO(x, h^i)) \wedge \neg TPP(x, h) \wedge (DC(x, y) \vee EC(x, y));$$

- $POut(x, h, h^i)$, read as "x is *partially outside* h with respect to entry boundary h^i," and equivalent to

$$\exists y\ H(h, y) \wedge EB(h^i, h) \wedge PO(x, h) \wedge PO(x, h^i) \wedge PO(x, \overline{\xi}) \wedge (DC(x, y) \vee EC(x, y));$$

  where $\overline{\xi}$ represents the complement of $x \cup y \cup h$.
- $JIn(x, h, h^i)$, read "x is *just inside* h with respect to boundary h^i," and equivalent to

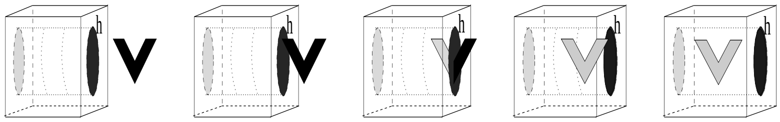
$$\exists y\ H(h, y) \wedge EB(h^i, h) \wedge (EC(x, h^i) \vee PO(x, h^i)) \wedge TPP(x, h) \wedge (DC(x, y) \vee EC(x, y));$$

- $WIn(x, h)$, read "x is *wholly inside* h," and equivalent to

$$PP(x, h) \wedge \neg\exists h^i (EB(h^i, h) \wedge C(x, h^i)).$$

$WOut$, $JOut$, $POut$, WIn and JIn are depicted in Figure 3, where the host object is the cuboid, the hole is the cylindrical figure inside the cuboid and the penetrating object is the **v**-shaped figure. Figure 3 can be understood as a sequence of continuous transitions from the relation *wholly outside* to *wholly inside*.

[2] In the case of an entry boundary (h^i) this is equivalent to assuming that h^i has a (infinitely) small fiat thickness.



(a) WOut (b) JOut (c) POut (d) JIn (e) WIn

Figure 3.: Relative location of an object **v** with respect to a hole h.

It is worth pointing out that, in contrast to (Casati and Varzi, 1999), encoding the relative location of an object with respect to a hole using RCC relations allowed us to include both $JOut$ and JIn into the same formalism since RCC is defined over the *closure* of regions. Therefore, the concepts of just inside and just outside can coexist with the initial assumption of holes as open regions. Another difference between the formalism presented above with respect to that proposed in (Casati and Varzi, 1999) is the inclusion of the hole entry boundary in the definitions of $JOut$, $POut$ and JIn, in order to account for the action of an object passing through a particular hole entry.

When a hole is crossed by a longer object, we may have combinations of these relations relative to the involved entry boundaries. Consider, for instance, the sequence of situations for inserting the string Str in the post hole Ph from its right boundary Ph^+ to its left boundary Ph^-, excluding the rest of the domain objects. We would move through the situations shown in Figure 4. In the automated solution we will present later, we will actually factor out all the intermediate situations between state 0 in Figure 4 (the string is wholly out of a hole) and state 4 (the string is crossing a hole through its two boundaries). Unfortunately, the current relations do not suffice to capture all the information within the puzzle‘s domain due to the flexibility of the string: it may be the case where several segments of the string are entangled inside a particular hole (without actually crossing it). Current predicates would just point out that the string is partially out the two hole boundaries, but not how many segments of the string are in the hole.

3 Crossings, segments and chains

As explained before, a crucial feature for a suitable puzzle representation is the possibility of dealing with the current crossings of a long object through the existing holes, bearing in mind that it can cross the same hole several times. Therefore, we can naturally think about a *crossing (region)* and try to formalise this concept using our spatial ontology. Formally, let l be a long object and h a hole. We say that the region c is a *crossing* between l and h, written $Crossing(c, h, l)$, if c is a proper part of l included in h that is passing through (exactly) two entry

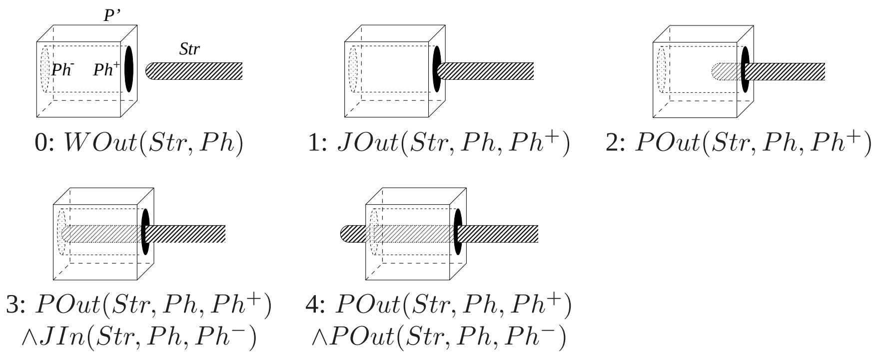


Figure 4.: Passing the string Str through the post hole Ph leftwards.

boundaries of h. More formally:

$$\begin{aligned} Crossing(c, l, h) \quad \equiv \quad & TPP(c, l) \wedge (TPP(c, h) \vee EQ(c, h)) \\ & \wedge \exists 2h^i \; Through(c, l, h, h^i) \\ & \wedge \neg \exists 3h^i \; Through(c, l, h, h^i) \end{aligned} \tag{4}$$

where $\exists nxP(x)$ with $n \geq 1$ means that there exist at least n different individuals x satisfying $P(x)$, that is:

$$\exists x_1, x_2, \ldots, x_n(\bigwedge_{i \in [1,n]} P(x_i) \wedge \bigwedge_{i,j \in [1,n], i \neq j} (x_i \neq x_j))$$

whereas predicate $Through(c, l, h, h^i)$ is defined as:

$$\begin{aligned} Through(c, l, h, h^i) \quad \equiv \quad & JIn(c, h, h^i) \wedge TPP(c, l) \\ & \wedge \exists y (TPP(y, l) \wedge EC(y, c) \wedge JOut(y, h, h^i)) \end{aligned}$$

or informally, region c just inside hole h wrt boundary h^i is externally connected to a subregion of l just outside h^i.

We should also guarantee that distinct hole crossings through the same hole do not overlap:

$$\begin{aligned} & Crossing(c, h, l) \wedge Crossing(c', h, l') \wedge c \neq c' \\ & \qquad \rightarrow DC(c, c') \vee EC(c, c') \end{aligned} \tag{5}$$

where l and l' might refer to the same long object. Note that the sort of long objects contains only non-perforated bodies. In cases where this constraint is not satisfied, Axiom 5 is not satisfied (take a co-axial cable passing through a hole as a counter example).

As it can be observed, crossings divide a long object as if it were partitioned by all the holes it is currently passing through. In the same way as all the crossings of l are proper parts of it, we can think about the remaining parts of l and call them *segments*. A *segment* can be defined as a maximal continuous part of a long object not overlapping with any hole. Formally, we begin defining:

$$\begin{aligned} Freepart(x,y) \quad \equiv \quad & (PP(x,y) \vee EQ(x,y)) \\ & \wedge \forall h[Hole(h) \rightarrow DC(x,h) \vee EC(x,h)] \end{aligned}$$

meaning that x is a part of y that does not overlap with any hole h. Then l_i is a segment of l, written $Segment(l_i, l)$ when it is a maximal free part of l, that is:

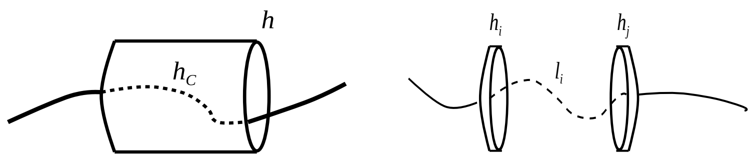
$$Segment(l_i, l) \quad \equiv \quad Freepart(l_i, l) \wedge \neg\exists x[PP(l_i, x) \wedge Freepart(x, l)]$$

Notice that the above definition of *segment* is general enough to cover objects with star shaped tips. For instance, we could perfectly capture in this way the segments of an Y-shaped body whose three arms are crossing different holes. In the puzzle, however, we deal with objects of a more restrictive shape, we have called *long objects*, that show as one of their essential feature that they allow us recognising a *linear* sequence of crossings from one of their tips to the other. Therefore, we include an additional axiom stating that, for any long object l, there cannot be a segment connecting more than two hole crossings:

$$\begin{aligned} Segment(l_i, l) \wedge Crossing(c, h, l) \wedge EC(l_i, c) \\ \wedge Crossing(c', h', l) \wedge EC(l_i, c') \\ \wedge Crossing(c'', h'', l) \wedge EC(l_i, c'') \\ \wedge c \neq c' \rightarrow c'' = c \vee c'' = c' \end{aligned}$$

Considering segments for other objects that do not show this "linear" feature is a topic for future research.

A graphical representation of hole crossings and long object segments is shown in Figure 5.



(a) Hole crossing h_c (b) long object segment l_i

Figure 5.: Hole crossing and long object segment.

We now use the concepts of hole crossing and long object segment to define a *chain* of crossings of a long object, something essential in our example domain to keep track of the string status at each situation. To define a chain for some long object x, we distinguish the two opposite extremities of x, denoted as x^- and x^+ (using a similar notation to that of hole boundaries) and respectively called *negative* and *positive terminals* of x. As a thumb rule, when not stated, we assume in all figures that the rightmost or the topmost extremities are positive, whereas the leftmost or the bottommost are negative (where the left-right dichotomy dominates the top-bottom one). For example, the right disk D_2 is linked to Str^+, while the post base B is linked to P^-. We, thus, associate each long object x to a list $chain(x)$ collecting the sequence of all hole crossings made by x following the direction from x^- to x^+. In the general case, where holes may have $n \geq 2$ entry boundaries, $chain(x)$ would collect a *pair of hole boundaries* (h^i, h^j) per each crossing, where h^i would be the "entrance" to the hole and h^j the "exit" in that imaginary travel from x^- to x^+. As an example, consider the string Str in Figure 6 crossing holes h, g and k, that correspond to the interiors of three hanging T-shirts. The list for $chain(Str)$ would correspond in this case to $[(h^1, h^3), (g^1, g^2), (k^2, k^4)]$.

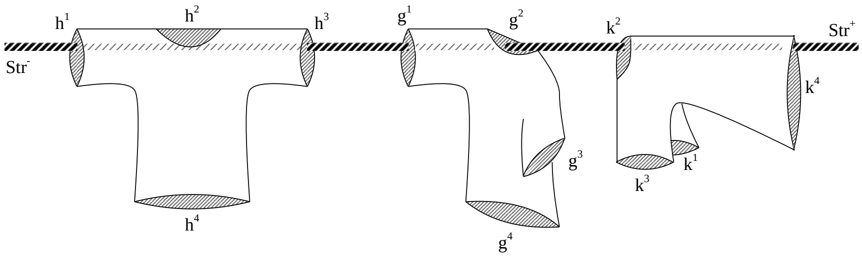
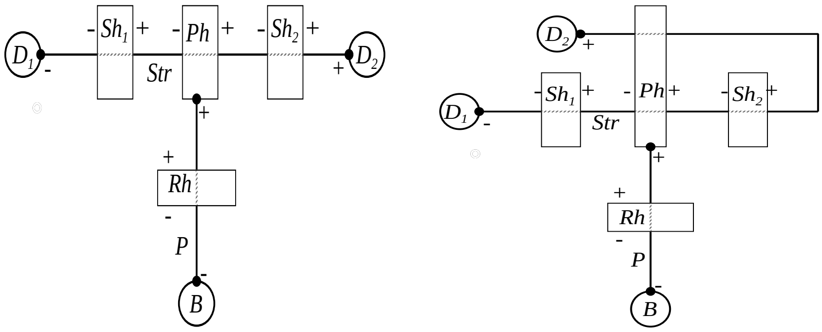


Figure 6.: An example of crossings as pairs of entry boundaries.

When all holes have $n = 2$ boundaries, however, we can simplify this notation by keeping only one boundary per crossing, say for instance, the hole exit. We illustrate the chain formalisation of the puzzle domain using diagrams. In these diagrams a box represents a *hole*, a circle a *regular object*, a thick line stands for a *long object* and a small black circle represents a link or connection. Examples of this graphical representation are shown in Figure 7. Following the previously defined criterion, $chain(Str)$ at situation $\mathcal{S}_0$ (Figure 7(a)) corresponds to the list of "outgoing" boundaries from Str^- to Str^+, that is, $chain(Str) = [Sh_1^+, Ph^+, Sh_2^+]$. Similarly, at $\mathcal{S}_1$ we would have $chain(Str) = [Sh_1^+, Ph^+, Sh_2^+, Ph^-]$.

In order to provide a formalisation of the list *chain* in first-order logic we define a function $next$ that will allow us to build the chain structure according to segments of long objects. To do that we assume an schematic representation of a chain as a linear graph whereby its nodes are segments and its edges are the hole crossings that connect two nodes (segments). The graph edges are labelled by the corresponding outgoing hole entry boundary. Therefore, the initial situation of the



(a) $\mathcal{S}_0$ (initial state) (b) $\mathcal{S}_1$ (D_2 passes left)

Figure 7.: States $\mathcal{S}_0$ and $\mathcal{S}_1$.

string Str (Figure 1(a)) could be represented as:

$$Str : 0 \xrightarrow{Sh_1^+} Str : 1 \xrightarrow{Ph^+} Str : 2 \xrightarrow{Sh_2^+} Str : 3, \quad (6)$$

where the numbers preceded by a colon in Formula 6 represent indexes enumerating the string segments.

The function $next$, then, takes a segment (and its index) and gives a tuple representing the next hole crossing and the index of the next segment (sweeping the long object from the negative to the positive tip). Therefore, $next(x, i) = \langle p, j\rangle$ represents the labelled graph edge $x : i \xrightarrow{p} x : j$. For convenience, the last segment in the list $x : i$ will further point to a special constant End, rather than to a pair $\langle p, j\rangle$.

The initial situation in the puzzle can be represented now by the formula:

$$\begin{aligned} next(Str, 0) = \langle Sh_1^+, 1\rangle \wedge next(Str, 1) = \langle Ph^+, 2\rangle \\ \wedge\, next(Str, 2) = \langle Sh_2^+, 3\rangle \wedge next(Str, 3) = End \\ \wedge\, next(P, 0) = \langle Rh^+, 1\rangle \wedge next(P, 1) = End \end{aligned} \quad (7)$$

It is easy to see from an inductive proof that the statement represented by Formula (7) is equivalent to the list description:

$$chain(Str) = [Sh_1^+, Ph^+, Sh_2^+] \wedge chain(P) = [Rh^+].$$

In what follows we use only the list representation of chains for brevity.

4 Acting on puzzle objects

To emphasise the utility of the $chain$ structure, and for completeness sake, we recall in this section the formal solution to the puzzle previously obtained in (Cabalar and Santos, 2006). The solution relies on two basic movements: the action $pass\_o$ (passing a long object terminal through a hole) and the action $pass\_h$ (passing a holed object through another hole).

The action $pass\_o(a, h^i)$ represents passing a long object tip a through some hole towards the hole boundary h^i. For example, the execution $pass\_o(Str^+, Ph^-)$ in the initial state $\mathcal{S}_0$ leads to $\mathcal{S}_1$ (both depicted in Figure 7) and corresponds to moving the positive terminal of Str (which is linked to disk D_2) to the left of the post hole (Ph^-).

It is clear that the execution of $pass\_o(x^+, h^i)$ (resp. $pass\_o(x^-, h^i)$) may equally mean that we are adding or removing the hole crossing from $chain(x)$ depending on the context. The possible effects of $pass\_o$ are depicted in Figure 8, where if B is h^+ (resp. h^-) then $-B$ stands for h^- (resp. h^+).

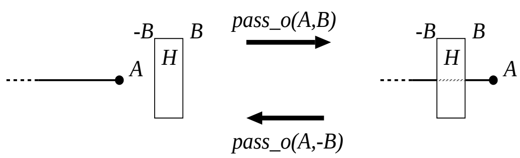


Figure 8.: Possible effects of $pass\_o$.

The action $pass\_h(a, b)$ represents passing a hole a towards a hole entry boundary b (belonging to a hole that is disjoint with a). Back to the example, we would execute the action $pass\_h(Ph, Rh^-)$ on the initial situation leading to the resulting state depicted in Figure 9.

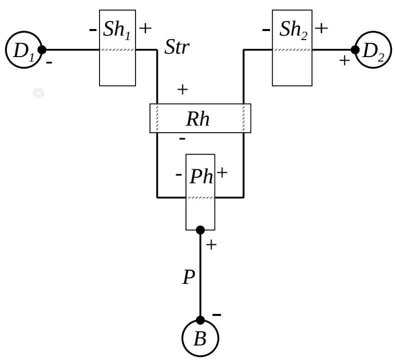


Figure 9.: Possible effect of $pass\_h$.

The most relevant effect of this action is that the string chain, which was previously unrelated to the ring hole, has gained two new crossings as an effect of $pass\_h(Ph, Rh^-)$. In other words, the list: $chain(Str) = [Sh_1^+, \underline{Ph^+}, Sh_2^+]$ has to be updated to: $chain(Str) = [Sh_1^+, \underline{Rh^-, Ph^+, Rh^+}, Sh_2^+]$. Note that this effect is due to the post-conditions of the action $pass\_h$ and not a consequence of the mereotopological description of the domain. This is because we just disregard the intermediate situations in which a holed object is crossing a hole and so, we are not considering their topological aspects either.

The complete set of possible movements is graphically shown in Figure 10. Formally, if we want to execute $pass\_h(x, e)$ and x is crossed by some string, then for any string Y crossing x, and any occurrence of x in $chain(Y)$, the list of possible movements would correspond to:

(1R) $chain(Y) = [\ldots, a, \underline{x^z}, b, \ldots] \Longrightarrow [\ldots, a, \underline{e, x^z, -e}, b, \ldots]$ with $a, b \notin \{e, -e\}$ or $a = e, b = -e$.

(1L) $chain(Y) = [\ldots, \underline{-e, x^z, e}, \ldots] \Longrightarrow [\ldots, \underline{x^z}, \ldots]$

(2R) $chain(Y) = [\ldots, a, \underline{x^z, e}, \ldots] \Longrightarrow [\ldots, a, \underline{e, x^z}, \ldots]$ with $a \neq -e$

(2L) $chain(Y) = [\ldots, \underline{-e, x^z}, a, \ldots] \Longrightarrow [\ldots, \underline{x^z, -e}, a, \ldots]$ with $a \neq e\}$

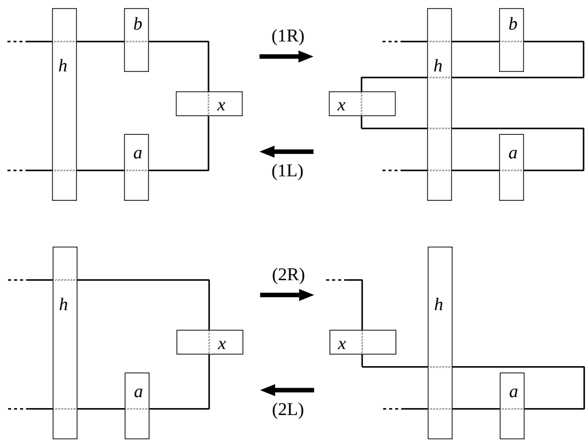


Figure 10.: Possible effects of $pass\_h$.

An important observation is that, while all the represented elements in each movement would be involved in the distinction of the movement type, only the underlined parts constitute the movement effect. This means, for instance, that in movement (2R), a is only used in the predecessor state to establish that we have a (2R) movement and not a (1L).

$$
\begin{array}{l}
0 : [Rh^+],\ [Sh_1^+, Ph^+, Sh_2^+] \\
\qquad\qquad\qquad pass\_o(Str^+, Ph^-) \\[1em]
1 : [Rh^+],\ [Sh_1^+, Ph^+, Sh_2^+, Ph^-] \\
\qquad\qquad\qquad pass\_o(P^+, Rh^-)\ \&\ pass\_h(Ph, Rh^-) \\[1em]
2 : [\,],\ [Sh_1^+, Rh^-, Ph^+, Rh^+, Sh_2^+, Rh^-, Ph^-, Rh^+] \\
\qquad\qquad\qquad pass\_h(Sh_2, Rh^-) \\[1em]
3 : [\,],\ [Sh_1^+, Rh^-, Ph^+, Sh_2^+, Ph^-, Rh^+] \\
\qquad\qquad\qquad pass\_h(Rh, Ph^+) \\[1em]
4 : [\,],\ [Sh_1^+, Ph^+, Rh^-, Sh_2^+, Rh^+, Ph^-] \\
\qquad\qquad\qquad pass\_h(Sh_2, Rh^+) \\[1em]
5 : [\,],\ [Sh_1^+, Ph^+, Sh_2^+, Ph^-]
\end{array}
$$

Figure 11.: A formal solution for the Fisherman's puzzle.

Using the chain description of the puzzle and the actions presented above a solution to the Fisherman's Folly puzzle can be represented by the sequence of chains shown on Figure 11 (and depicted in Figure 12), whereby each state is identified by its sequence number plus the pair of lists $chain(P)$ and $chain(Str)$ in this order. The performed actions in each transition are interlaced between each state i and the next one $i+1$. Note that State 5 has actually reached the goal since, at this point, the ring hole Rh does not occur in any list, i.e., it is not crossed by any long object[3].

5 Concluding remarks

In this work we investigated knowledge representation issues regarding the spatial aspects of a puzzle. The puzzle chosen is called Fisherman's Folly and is constituted by an arrangement of rigid objects and non-trivial elements such as holes and a string. This paper defined the basic elements of an ontology about the puzzle using a mereotopological theory about holes (following the guidelines in (Casati and Varzi, 1999)).

We also proposed a novel data structure, called *chain*, for representing the arrangement of the domain objects. Using this data structure, allied with the def-

[3]Remember that the goal of the puzzle is to free the ring from the system of objects, while the configuration left for the rest of objects is irrelevant.

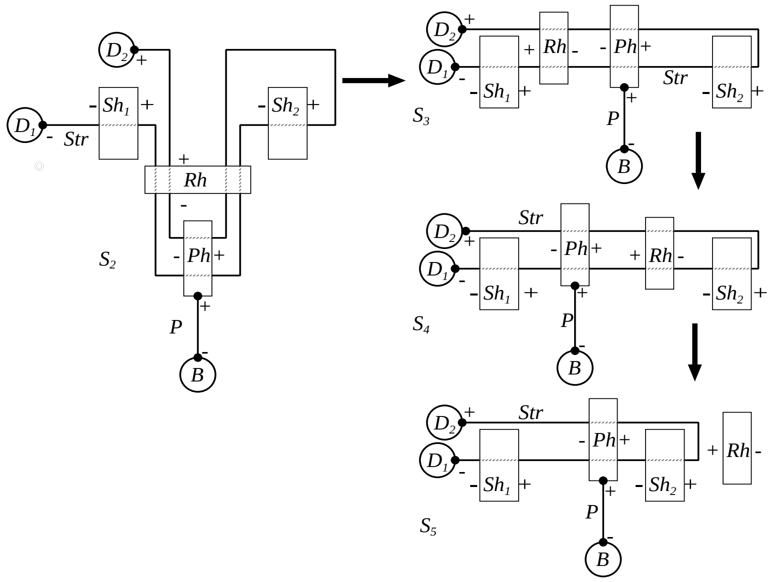


Figure 12.: The diagrams for the solution presented in Figure 11. The states S_0 and S_1 are shown in Figure 7.

initions of two actions, we presented a formal solution to the Fisherman's Folly puzzle (whose automated reasoning implementation has been presented elsewhere (Cabalar and Santos, 2006)). In our current investigations we have been successfully using similar techniques to provide automated solutions to various other puzzles composed of strings, holes and solid objects (Cabalar and Santos, 2007). These findings suggest that the results of the present investigation may be used in a variety of domains where there is an interplay between flexible objects (such as strings) and immaterial bodies (such as holes).

The present paper is, however, only a first step towards a complete understanding of how to represent and reason about complex arrangements of non-trivial objects that include flexibility or penetrability as some of their spatial attributes. The region-based ontology we propose is an attempt to provide a rigorous account for the domain objects so that properties about space could be proved in this context. Nevertheless, our interest in the ontology was to define a formalism about actions and change on spatial knowledge that is capable of solving puzzles such as that presented in this paper. The success of the present paper on this endeavour was partial. On the one hand, the mereotopological theory (that was the starting point of this work) was rich enough to allow the definition of *hole crossing*, *string segment* and *chain* (Section 3). On the other hand, the actions facilitating an autonomous solution to the puzzle had to be built outside the starting mereotopological definitions. Therefore, the problem of defining the concepts of actions and change within a spatial domain remains still open.

Another open issue for future research is the consideration of knots in the domain and the investigation of the consequent increase in the complexity of problem solving due to the multitude of states that knots imply.

The ultimate goal of this research is to provide a rigorous account of reasoning about flexible objects, immaterial bodies and actions involving them in order to facilitate automated reasoning about commonsense spatial knowledge.

Acknowledgements: Paulo Santos acknowledges support from FAPESP, São Paulo, Brazil; Pedro Cabalar is partially supported by Spanish Ministry MEC project TIN2006-15455-C03-02, Spain.

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