P-log: Probabilistic Reasoning with Answer Set Programming

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1 Introduction

2 P-log
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This chapter is based on:


Probability Theory: a well studied and developed branch of Mathematics.
Motivation

- **Probability Theory**: a well studied and developed branch of Mathematics.

- However, its basic notions are *not always intuitive* for commonsense reasoning.

- This may make Classical Probability Theory alone *not suitable for KR.*
Motivation

- **Probability Theory**: a well studied and developed branch of Mathematics.
- However, its basic notions are *not always intuitive* for commonsense reasoning.
- This may make Classical Probability Theory alone *not suitable for KR*. Let us see a pair of examples.
Example 1. Monty Hall problem

- A player is given the choice to select 1 of 3 closed doors. One of them has a prize and the other two are empty.
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- The TV show conductor, Monty, \textit{always opens an empty room}. Then, he lets the player switch if he likes.
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- A player is given the choice to select 1 of 3 closed doors. One of them has a prize and the other two are empty.

- The TV show conductor, Monty, always opens an empty room. Then, he lets the player switch if he likes. Does it really matter?
Example 1. Monty Hall problem

- A player is given the choice to select 1 of 3 closed doors. One of them has a prize and the other two are empty.

- It does: he should switch because he has the double chances!!
Example 2. Simpson’s Paradox

Recovery rates for a drug treatment observed among males and females

Males:

\[
\begin{array}{ccc}
\text{fraction\_of\_population} & \text{recovery\_rate} \\
\text{drug} & 3/8 & 60\% \\
\text{-drug} & 1/8 & 70\%
\end{array}
\]

Females:

\[
\begin{array}{ccc}
\text{fraction\_of\_population} & \text{recovery\_rate} \\
\text{drug} & 1/8 & 20\% \\
\text{-drug} & 3/8 & 30\%
\end{array}
\]

A patient $P$ consults the doctor about trying the drug.
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If \( P \) is male, the advice is not to take the drug:

\[
0.7 = P(\text{recover} \mid \text{male}, \neg\text{drug}) \not< P(\text{recover} \mid \text{male, drug}) = 0.6
\]
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If $P$ is female, the advice is not to take the drug either:

$$0.3 = P(\text{recover} \mid \text{female}, \neg\text{drug}) \nless\ than\ than\ P(\text{recover} \mid \text{female}, \text{drug}) = 0.2$$
## Example 2. Simpson’s Paradox

Recovery rates for a drug treatment observed among males and females

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If $P$’s sex is unknown . . .

$$??? = P(\text{recover, } \neg \text{drug}) \quad P(\text{recover, } \text{drug}) = ???$$
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If *P*’s sex is **unknown** . . . the advice is **taking** the drug ?!?

\[
0.4 = P(\text{recover}, \neg \text{drug}) < P(\text{recover}, \text{drug}) = 0.5
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Example 2. Simpson’s Paradox

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- The problem has to do with **causal direction** between variables.
- Probability Theory cannot tell whether recovery can be an effect of **drug** or vice versa (i.e., when they do not recover, they take the drug, for instance).
- Judea Pearl’s **causal networks** are a variant of Bayesian networks where we can cut a link by **causal intervention**:

\[ P(\text{recover} \mid \text{do}(<\text{drug}>)) \]

means probability of recovering when we fix **drug**.
Probabilistic Reasoning


commonsense reasoning about the degree of an agent’s belief in the likelihood of different events
Probabilistic Reasoning


  *commonsense reasoning about the degree of an agent’s belief in the likelihood of different events*

- An illustrative example: *lost in the jungle* you are captured by natives that will help you to survive if you (blindly) extract a stone from an urn of the color, black or white, you previously select.

  You are told that 9 stones are white and one is black.

  Which color should you tell?
Probabilistic models

Suppose we enumerate the stones 1, \ldots, 10 and stone 1 is the black one.
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$$\sum_{W \in \Omega} \mu(W) = 1$$

3. $P : 2^\Omega \mapsto [0, 1]$ probability function

$$P(E) = \sum_{W \in E} \mu(W)$$

We can also use a formula $F$: $P(F) = P(\{W \mid W \models F\})$
Probabilistic models

In the example $W_1 = \{ \text{selcolor = white, draw = 1, } \neg \text{help} \}$, $W_2 = \{ \text{selcolor = white, draw = 2, help} \}$ ...

Principle of indifference: under no preference, possible outcomes of a random experiment are equally probable.

Therefore $\mu(W_i) = \frac{1}{10}$ for $i = 1, \ldots, 10$.

Suppose we select white, then $P(\text{help}) = \frac{9}{10}$. If we select black instead, we get $P(\text{help}) = \frac{1}{10}$. 
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P. Cabalar (Depto. Computación, University of Corunna, SPAIN)
P-log [Baral, Gelfond, Rushton 09]: main idea

possible worlds = answer sets of a (probabilistic) logic program.
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- The syntax is close to ASP. We allow atoms of the form $a(t) = v$
  where $a$ is a functional attribute, $t$ a tuple of terms and $v$ a value in
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For Boolean attributes we may use $a(t)$ and $\neg a(t)$ to stand for $a(t) = true$ and $a(t) = false$, respectively.
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$a(t) = true$ and $a(t) = false$, respectively.

A P-log program includes additional probabilistic constructs we
will see next.
The jungle example in P-log

% sorts and general variables
stones={1..10}.
colors={black,white}.
boolean={true,false}.
#domain stones(X).

% Setting the color of each stone
color: stone -> colors.
color(1)=black.
color(X)=white :- X<>1.

% Other attributes
selcolor:colors. % selected color
help:boolean.
% Random variable draw = number of the picked stone
draw:stones.
[r] random(draw).

% Representing the tribal laws
help=true :- draw=X, color(X)=C, selcolor=C.
help=false :- draw=X, color(X)=C, selcolor<>C.

% Suppose we chose white
selcolor=white.

% And we ask the probability of getting help
? {help=true}.
The jungle example in P-log

- We make the call
  
  \texttt{plog \-t jungle.txt}

  and obtain a probability of 0.9.
The jungle example in P-log

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  plog -t jungle.txt

  and obtain a probability of 0.9.

- Try with selcolor=black instead.
Causal probability statements

- The indifference principle has set all outcomes equally probable, but we can fix probabilities.

Suppose that, when you select color white, stones are introduced in an irregular urn so that, due to the stone shapes, the probability of picking the black stone is $\frac{1}{3}$. We add the statement:

$$\text{pr(draw}=1|\text{selcolor}=\text{white})=\frac{1}{3}.$$ 

Which is the probability of getting help now when selecting white? and when we select black? Compute the probability of picking stone 2 in both cases. When selcolor = white the rest of stones are equally probable ($1 - \frac{1}{3} = \frac{2}{3} = \frac{2}{27} = 0.074$).
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\[
[r] \quad \text{pr(draw=1|selcolor=white)} = \frac{1}{3}.
\]

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- We add the statement:
  \[ r \] \( pr(draw=1|selcolor=white)=1/3 \).
- Which is the probability of getting help now when selecting white and when we select black?
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  \[(1 - 1/3)/9 = 2/27 = 0.074\]
Observations and interventions

- P-log allows declaring **observations** as follows:
  \[ \text{obs}(a(t)=v). \]
  meaning that we rule out worlds where \( a(t)=v \) does not hold.

- We can also declare **interventions** as follows:
  \[ \text{do}(a(t)=v). \]
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Observations and interventions

To illustrate the difference, take Simpson’s paradox scenario:

boolean = \{t, f\}.

male, recover, drug : boolean.

[r1] random(male).
[r2] random(recover).
[r3] random(drug).

[r1] pr(male = t)=1/2.
[r2] pr(recover = t | male = t, drug = t) =3/5.
[r2] pr(recover = t | male = t, drug = f)=7/10.
[r2] pr(recover = t | male = f, drug = t)=1/5.
[r2] pr(recover = t | male = f, drug = f)=3/10.
[r3] pr(drug = t | male = t)=3/4.
[r3] pr(drug = t | male = f)=1/4.
Observations and interventions

- Try, one by one, the following queries:

  ?{recover=t} | do (drug=t).
  ?{recover=t} | do (drug=f).
  ?{recover=t} | obs (drug=t).
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  - `?{recover=t} | obs(drug=t).`
  - `?{recover=t} | obs(drug=f).`

- Using causal interventions yields the expected result (we shouldn’t take the drug).

- Using just observations leads to (what seemed a) paradox.
The Monty Hall problem in P-log

- The solution to Monty Hall problem is quite simple. It suffices with limiting the random values that Monty can play with.

```prolog
#domain doors(D).
boolean={true, false}. doors={1,2,3}.
prize,open,selected:doors.

can_open: doors -> boolean.
can_open(D)=false:- selected=D.
can_open(D)=false:- prize=D.
can_open(D)=true:- not can_open(D)=false.

[r1] random (prize).
[r3] random (open:{X:can_open(X)}).
[r2] random (selected).
?{prize=3}|obs(selected=1),obs(open=2).
```