Towards Dynamic Answer Set Programming over finite traces

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Abstract. Our ultimate goal is to conceive an extension of Answer Set Programming with language constructs from dynamic (and temporal) logic to provide an expressive computational framework for modeling dynamic applications. To address this in a semantically well founded way, we generalize the definition of Dynamic Equilibrium Logic to accommodate finite linear time and extend it with a converse operator in order to capture past temporal operators. This results in a general logical framework integrating existing dynamic and temporal logics of Here-and-There over both finite and infinite time. In the context of finite time, we then develop a translation of dynamic formulas into propositional ones that can in turn be translated into logic programs.

1 Introduction

Answer Set Programming (ASP [13]) has become a popular approach to solving knowledge-intense combinatorial search problems due to its performant solving engines and expressive modeling language. However, both are mainly geared towards static domains and lack native support for handling dynamic applications. We have addressed this shortcoming over the last decade by creating a temporal extension of ASP [1] based on Linear Temporal Logic (LTL [15]) that has recently led to the temporal ASP system *telingo* [5]. The approach of LTL has however its limitations when it comes to expressing control over temporal trajectories. Such control can be better addressed with Dynamic Logic (DL [16]), offering a more fine-grained approach to temporal reasoning thanks to the possibility to form complex actions from primitive ones.[4] To this end, DL relies on modal propositions, like $[\rho]\,\varphi$, to express that all executions of (complex) action ρ terminate in a state satisfying φ. As an example, consider a "Russian roulette" variation of the Yale-shooting-scenario, so the turkey is dead after we pull the trigger as many times as needed until we reach the loaded chamber. This can be expressed in DL via the proposition: $[\mathbf{while}\ \neg \mathit{loaded}\ \mathbf{do}\ \mathit{trigger}; \mathit{trigger}]\ \mathit{dead}$. The term within brackets delineates trajectories matching the regular expression '$(\neg \mathit{loaded}?; \mathit{trigger})^*; \mathit{loaded}?; \mathit{trigger}$', where $\varphi?$ tests whether φ holds at the state at hand, and ';' and '*' are the sequential

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[4] The same consideration led to GOLOG [12] in the context of the situation calculus.

composition and iteration operators, respectively. With this, the above proposition is satisfied whenever the state following a matching trajectory entails $dead$.

This expressive power motivated us to introduce the basic foundations of an extension of ASP with dynamic operators from DL in [4]. In what follows, we build upon these foundations (i) to introduce a general logical framework comprising previous dynamic and temporal extensions and (ii) to elaborate upon a translation to propositional theories that can in turn be compiled into logic programs. To this end, we follow the good practice of first introducing an extension to ASP's base logic, the *Logic of Here-and-There* (HT [11]), and then to devise an appropriate reduction. An HT interpretation $\langle H, T\rangle$ is a pair of interpretations that can be seen as being three-valued, where atoms in H are "*certainly true*," atoms not in T are "*false*" and atoms in T are "*potentially true*." This explains the usual condition $H \subseteq T$, meaning that anything certainly true is also potentially true. An HT interpretation $\langle H, T\rangle$ is said to be *total* if $H = T$, that is, the mapping becomes two-valued. Total interpretations satisfying a certain minimality condition are known to correspond to stable models; they are also referred to as equilibrium models, and the resulting logic is called *Equilibrium Logic* (EL). For capturing (linear) time, sequences of such HT interpretations are considered, similar to LTL. In accord with [7], we argue that such linear traces provide an appropriate semantic account of time in our context, and thus base also our dynamic extension of ASP on the same kind of semantic structures.

Our ultimate goal is to conceive an extension of ASP with language constructs from dynamic (and temporal) logic in order to provide an expressive computational framework for modeling dynamic applications. To address this in a semantically well founded way, we generalize the definition of *Dynamic* HT *and* EL (DHT/DEL [4]) to accommodate finite traces and augment it with a converse operator (in order to capture past temporal operators). This not only allows us to embed temporal extensions of ASP, such as *Temporal Equilibrium Logic* over finite traces (TEL_f [5]) along with its past and future operators, and more standard ones like LTL_f [7], but moreover provides us with blueprints for implementation on top of existing (temporal) ASP solvers like *telingo*. Indeed, DEL_f can be regarded as a non-monotonic counterpart of LTL_f [7], being in an analgous relationship as classical and equilibrium logic, or SAT and ASP, respectively.

More precisely, we start in Section 2 by defining a general logical framework integrating existing dynamic and temporal logics of Here-and-There and their associated Equilibrium logics over both finite and infinite traces. Section 3 is dedicated to the computational development of our approach in the context of finite traces. We introduce a translation from dynamic formulas to propositional ones by relying on a normal form for complex actions. Finally, Section 4 concludes the paper.

2 Linear Dynamic Equilibrium Logic

Given a set $\mathcal{A}$ of propositional variables (called *alphabet*), *dynamic formulas* φ and *path expressions* ρ are mutually defined as in [7] by the pair of grammar rules:

$$\varphi ::= a \mid \bot \mid \top \mid [\rho]\,\varphi \mid \langle\rho\rangle\,\varphi\ , \qquad\qquad \rho ::= \top \mid \varphi? \mid \rho + \rho \mid \rho\ ;\ \rho \mid \rho^* \mid \rho^-\ .$$

Each ρ is a regular expression formed with the truth constant $\top$ plus the test construct $\varphi?$ typical of Dynamic Logic (DL [9]). An important feature that departs from DL is that, in the latter, atomic path expressions are formed with a sort of so-called *atomic actions* that is separated from propositional atoms in $\mathcal{A}$, used for formulas. We adopt the approach of [7] and considering that the only atomic path expression is $\top$, keeping the test construct $\varphi?$ that may refer to propositional atoms in the (single) alphabet $\mathcal{A}$.

As we show further below, the above language allows us to capture several derived operators, like the Boolean and temporal ones:

$$
\begin{array}{rclrcl}
\varphi \wedge \psi & \stackrel{def}{=} & \langle \varphi? \rangle\, \psi & \varphi \vee \psi & \stackrel{def}{=} & \langle \varphi? + \psi? \rangle\, \top \\
\varphi \to \psi & \stackrel{def}{=} & [\varphi?]\, \psi & \neg\varphi & \stackrel{def}{=} & \varphi \to \bot \\
\mathbb{F} & \stackrel{def}{=} & [\top]\bot & \mathbf{I} & \stackrel{def}{=} & [\top^-]\bot \\
\circ\varphi & \stackrel{def}{=} & \langle \top \rangle\, \varphi & \bullet\varphi & \stackrel{def}{=} & \langle \top^- \rangle\, \varphi \\
\widehat{\circ}\varphi & \stackrel{def}{=} & [\top]\, \varphi & \widehat{\bullet}\varphi & \stackrel{def}{=} & [\top^-]\, \varphi \\
\Diamond\varphi & \stackrel{def}{=} & \langle \top^* \rangle\, \varphi & \blacklozenge\varphi & \stackrel{def}{=} & \langle \top^{*-} \rangle\, \varphi \\
\Box\varphi & \stackrel{def}{=} & [\top^*]\, \varphi & \blacksquare\varphi & \stackrel{def}{=} & [\top^{*-}]\, \varphi \\
\varphi\, \mathbb{U}\, \psi & \stackrel{def}{=} & \langle (\varphi?; \top)^* \rangle\, \psi & \varphi\, \mathbf{S}\, \psi & \stackrel{def}{=} & \langle (\varphi?; \top)^{*-} \rangle\, \psi \\
\varphi\, \mathbb{R}\, \psi & \stackrel{def}{=} & (\psi\, \mathbb{U}\, (\varphi \wedge \psi)) \vee \Box\psi & \varphi\, \mathbf{T}\, \psi & \stackrel{def}{=} & (\psi\, \mathbf{S}\, (\varphi \wedge \psi)) \vee \blacksquare\psi
\end{array}
$$

While negation $\neg$ is expressed as usual in HT via implication, all other connectives are defined in terms of the dynamic operators $\langle\cdot\rangle$ and $[\cdot]$. This involves the Booleans' $\wedge$, $\vee$, and $\to$, among which the definition of $\to$ is most noteworthy since it hints at the implicative nature of $[\cdot]$, as well as the future temporal operators $\mathbb{F}$, $\circ$, $\widehat{\circ}$, $\Diamond$, $\Box$, $\mathbb{U}$, $\mathbb{R}$, standing for *final, next, weak next, eventually, always, until,* and *release*, and their past-oriented counterparts: $\mathbf{I}$, $\bullet$, $\widehat{\bullet}$, $\blacklozenge$, $\blacksquare$, $\mathbf{S}$, $\mathbf{T}$. The weak one-step operators, $\widehat{\circ}$ and $\widehat{\bullet}$, are of particular interest when dealing with finite traces, since their behavior differs from their genuine counterparts only at the ends of a trace. In fact, $\widehat{\circ}\varphi$ can also be expressed as $\circ\varphi \vee \mathbb{F}$ (and $\widehat{\bullet}$ as $\bullet\varphi \vee \mathbf{I}$). Finally, note that the converse operator ρ^- is essential for expressing all temporal past operators, whose addition in temporal logic is exponentially more succinct than using only future operators [2]. A formula is *propositional*, if all its connectives are Boolean, and *temporal*, if it includes only Boolean and temporal ones. A dynamic formula is said to be *conditional* if it contains some occurrence of an atom $p \in \mathcal{A}$ inside a $[\cdot]$ operator; it is called *unconditional* otherwise. Note that formulas with atoms in implication antecedents or negated formulas are also conditional, since they are derived from $[\cdot]$. For instance, $[p?]\,\bot$ is conditional, and is actually the same as $p \to \bot$ and $\neg p$. As usual, a *(dynamic) theory* is a set of (dynamic) formulas.

Following the definition of *linear* DL (LDL) in [7], we sometimes use a propositional formula ϕ as a path expression actually standing for $(\phi?; \top)$. Another abbreviation is the sequence of n repetitions of some expression ρ defined as $\rho^0 \stackrel{def}{=} \top?$ and $\rho^{n+1} \stackrel{def}{=} \rho; \rho^n$. For instance, $\rho^3 = \rho; \rho; \rho; \top?$ which amounts to $\rho; \rho; \rho$, as we see below.

Given $a \in \mathbb{N}$ and $b \in \mathbb{N} \cup \{\omega\}$, we let $[a..b]$ stand for the set $\{i \in \mathbb{N} \mid a \leq i \leq b\}$ and $[a..b)$ for $\{i \in \mathbb{N} \mid a \leq i < b\}$. For the semantics, we start by defining a *trace* of length λ over alphabet $\mathcal{A}$ as a sequence $\langle H_i \rangle_{i \in [0..\lambda)}$ of sets $H_i \subseteq \mathcal{A}$. A trace is *infinite* if $\lambda = \omega$ and *finite* otherwise, that is, $\lambda = n$ for some natural number $n \in \mathbb{N}$. Given traces $\mathbf{H} = \langle H_i \rangle_{i \in [0..\lambda)}$ and $\mathbf{H}' = \langle H'_i \rangle_{i \in [0..\lambda)}$ both of length λ, we write $\mathbf{H} \leq \mathbf{H}'$ if $H_i \subseteq H'_i$ for each $i \in [0..\lambda)$; accordingly, $\mathbf{H} < \mathbf{H}'$ iff both $\mathbf{H} \leq \mathbf{H}'$ and $\mathbf{H} \neq \mathbf{H}'$.

A *Here-and-There trace* (for short HT-*trace*) of length λ over alphabet $\mathcal{A}$ is a sequence of pairs $\langle H_i, T_i \rangle_{i \in [0..\lambda)}$ such that $H_i \subseteq T_i \subseteq \mathcal{A}$ for any $i \in [0..\lambda)$. As before, an HT-trace is infinite if $\lambda = \omega$ and finite otherwise. We often represent an HT-trace as a pair of traces $\langle \mathbf{H}, \mathbf{T} \rangle$ of length λ where $\mathbf{H} = \langle H_i \rangle_{i \in [0..\lambda)}$ and $\mathbf{T} = \langle T_i \rangle_{i \in [0..\lambda)}$ and $\mathbf{H} \leq \mathbf{T}$. A particular type of HT-traces satisfy $\mathbf{H} = \mathbf{T}$ and are called *total*.

We proceed by generalizing the extension of HT with dynamic operators, called DHT in [4], to HT-traces of fixed length in order to integrate finite as well as infinite traces, and by adding the converse operator. The overall definition of DHT satisfaction relies on a double induction. Given any HT-trace $\mathbf{M} = \langle \mathbf{H}, \mathbf{T} \rangle$, we define DHT satisfaction of formulas, $\mathbf{M}, k \models \varphi$, in terms of an accessibility relation for path expressions $\|\rho\|^{\mathbf{M}} \subseteq \mathbb{N}^2$ whose extent depends again on $\models$.

Definition 1 (DHT **satisfaction).** *An* HT-*trace* $\mathbf{M} = \langle \mathbf{H}, \mathbf{T} \rangle$ *of length* λ *over alphabet* $\mathcal{A}$ satisfies *a dynamic formula* φ *at time point* $k \in [0..\lambda)$*, written* $\mathbf{M}, k \models \varphi$*, if the following conditions hold:*

1. $\mathbf{M}, k \models \top$ *and* $\mathbf{M}, k \not\models \bot$
2. $\mathbf{M}, k \models a$ *if* $a \in H_k$ *for any atom* $a \in \mathcal{A}$
3. $\mathbf{M}, k \models \langle \rho \rangle\, \varphi$ *if* $\mathbf{M}, i \models \varphi$ *for some* i *with* $(k, i) \in \|\rho\|^{\mathbf{M}}$
4. $\mathbf{M}, k \models [\rho]\, \varphi$ *if* $\mathbf{M}', i \models \varphi$ *for all* i *with* $(k, i) \in \|\rho\|^{\mathbf{M}'}$ *for both* $\mathbf{M}' = \mathbf{M}$ *and* $\mathbf{M}' = \langle \mathbf{T}, \mathbf{T} \rangle$

where, for any HT-*trace* $\mathbf{M}$*,* $\|\rho\|^{\mathbf{M}} \subseteq \mathbb{N}^2$ *is a relation on pairs of time points inductively defined as follows.*

5. $\|\top\|^{\mathbf{M}} \stackrel{def}{=} \{(i, i+1) \mid i, i+1 \in [0..\lambda)\}$
6. $\|\varphi?\|^{\mathbf{M}} \stackrel{def}{=} \{(i, i) \mid \mathbf{M}, i \models \varphi\}$
7. $\|\rho_1 + \rho_2\|^{\mathbf{M}} \stackrel{def}{=} \|\rho_2\|^{\mathbf{M}} \cup \|\rho_2\|^{\mathbf{M}}$
8. $\|\rho_1 \,;\, \rho_2\|^{\mathbf{M}} \stackrel{def}{=} \{(i, j) \mid (i, k) \in \|\rho_1\|^{\mathbf{M}}$ *and* $(k, j) \in \|\rho_2\|^{\mathbf{M}}$ *for some* $k\}$
9. $\|\rho^*\|^{\mathbf{M}} \stackrel{def}{=} \bigcup_{n \geq 0} \|\rho^n\|^{\mathbf{M}}$
10. $\|\rho^-\|^{\mathbf{M}} \stackrel{def}{=} \{(i, j) \mid (j, i) \in \|\rho\|^{\mathbf{M}}\}$ □

The following properties can be easily observed by inspection of the semantics.

Proposition 1. *Relation* $\|\rho\|^{\mathbf{M}}$ *defined above satisfies* $\|\rho\|^{\mathbf{M}} \subseteq [0..\lambda) \times [0..\lambda)$. □

Proposition 2. *If* ρ *is converse-free and* $(i, j) \in \|\rho\|^{\mathbf{M}}$ *then* $i \leq j$. □

As we can see, $\langle \rho \rangle\, \varphi$ and $[\rho]\, \varphi$ quantify over time points i that are reachable under path expression ρ at the current point k, that is, $(k, i) \in \|\rho\|^{\mathbf{M}}$. The main difference with respect to [4] is that $\|\rho\|^{\mathbf{M}} \subseteq [0..\lambda) \times [0..\lambda)$ so that all pairs in that relation are now confined to the set of defined time points $[0..\lambda)$. This additional restriction is due to two reasons. First, it is now possible to access time points in the past $i < k$ using the converse operator ρ^-, something impossible with the converse-free path expressions in [4]. As a result, we must restrict $i \geq 0$ to avoid going backwards, further than the initial situation. Second, for a similar reason, when we have a finite length $\lambda = n$, we must also impose $i < n$, something not needed for infinite traces $\lambda = \omega$ since any natural number obviously satisfies $i < \omega$.

An HT-trace $\mathbf{M}$ is a *model* of a dynamic theory Γ if $\mathbf{M}, 0 \models \varphi$ for all $\varphi \in \Gamma$. We write $\mathrm{DHT}(\Gamma, \lambda)$ to stand for the set of DHT models of length λ of a theory Γ, and define $\mathrm{DHT}(\Gamma) \stackrel{def}{=} \bigcup_{\lambda=0}^{\omega} \mathrm{DHT}(\Gamma, \lambda)$, that is, the whole set of models of Γ of any length. When $\Gamma = \{\varphi\}$ we just write $\mathrm{DHT}(\varphi, \lambda)$ and $\mathrm{DHT}(\varphi)$.

A formula φ is a *tautology* (or is *valid*), written $\models \varphi$, iff $\mathbf{M}, k \models \varphi$ for any HT-trace and any $k \in [0..\lambda)$. We call the logic induced by the set of all tautologies *(Linear) Dynamic logic of Here-and-There* (DHT for short). Two formulas φ, ψ are said to be *equivalent*, written $\varphi \equiv \psi$, whenever $\mathbf{M}, k \models \varphi$ iff $\mathbf{M}, k \models \psi$ for any HT-trace $\mathbf{M}$ and any $k \in [0..\lambda)$. This allows us to replace φ by ψ and vice versa in any context, and is the same as requiring that $\varphi \leftrightarrow \psi$ is a tautology. Note that this relation, $\varphi \equiv \psi$, is stronger than coincidence of models $\mathrm{DHT}(\varphi) = \mathrm{DHT}(\psi)$. For instance, $\mathrm{DHT}(\bullet\top) = \mathrm{DHT}(\langle \top^- \rangle\, \top) = \emptyset$ because models are checked at the initial situation $k = 0$ and there is no previous situation at that point, so $\mathrm{DHT}(\bullet\top) = \mathrm{DHT}(\bot)$. However, in general, $\bullet\top \not\equiv \bot$ since $\bullet\top$ is satisfied for any $k > 0$ (for instance $\circ\bullet\top \not\equiv \circ\bot$ but $\circ\bullet\top \equiv \top$ instead).

As with formulas, we say that path expressions ρ_1, ρ_2 are *equivalent*, written $\rho_1 = \rho_2$, when $\|\rho_1\|^{\mathbf{M}} = \|\rho_2\|^{\mathbf{M}}$ for any HT-trace $\mathbf{M}$. For instance, it is easy to see that:

$$(\rho_1; \rho_2); \rho_3 = \rho_1; (\rho_2; \rho_3) \qquad \rho^* = \top? + (\rho; \rho^*)$$
$$\top?; \rho = \rho; \top? = \rho \qquad \rho; \rho^* = \rho^*; \rho$$

The following equivalences of path expressions allow us to push the converse operator inside, until it is only applied to $\top$.

Proposition 3. *For all path expressions ρ_1, ρ_2 and ρ and for all formulas φ, the following equivalences hold:*

$$(\rho^-)^- = \rho \qquad (\varphi?)^- = \varphi? \qquad (\rho^*)^- = (\rho^-)^*$$
$$(\rho_1 + \rho_2)^- = \rho_1^- + \rho_2^- \qquad (\rho_1; \rho_2)^- = \rho_2^-; \rho_1^-$$

We prove next that a pair of basic properties from HT already satisfied in [4] are maintained in the current extension of DHT.

Proposition 4 (Persistence). *For any* HT*-trace $\langle \mathbf{H}, \mathbf{T} \rangle$ of length λ, any dynamic formula φ and any path expression ρ, we have:*

1. $\langle \mathbf{H}, \mathbf{T} \rangle, k \models \varphi$ *implies* $\langle \mathbf{T}, \mathbf{T} \rangle, k \models \varphi$*, for all* $k \in [0..\lambda)$
2. $\|\rho\|^{\langle \mathbf{H}, \mathbf{T} \rangle} \subseteq \|\rho\|^{\langle \mathbf{T}, \mathbf{T} \rangle}$. □

Persistence is a property known from intuitionistic logic; it expresses that accessible worlds satisfy the same or more formulas than the current world, where $\mathbf{T}$ is "accessible" from $\mathbf{H}$ in HT. This also explains the semantics of $[\rho]\,\varphi$, which behaves as a kind of intuitionistic implication (used to define '$\rightarrow$' as a derived operator) and so, it must hold for all accessible worlds, viz. $\langle \mathbf{H}, \mathbf{T} \rangle$ and $\langle \mathbf{T}, \mathbf{T} \rangle$.

For simplicity, we refrain from introducing the semantics of LDL [7], since it just corresponds to DHT on total traces $\langle \mathbf{T}, \mathbf{T} \rangle$, as stated below. Let us simply use $\mathbf{T}, k \models \varphi$ to denote the satisfaction of φ by a trace $\mathbf{T}$ at point k in LDL and $\|\rho\|^{\mathbf{T}}$ the LDL accessibility relation for ρ and $\mathbf{T}$.

Proposition 5. *For any total* HT*-trace* $\langle \mathbf{T}, \mathbf{T} \rangle$ *of length* λ*, any dynamic formula* φ *and any path expression* ρ*, we have: (1)* $\langle \mathbf{T}, \mathbf{T} \rangle, k \models \varphi$ *iff* $\mathbf{T}, k \models \varphi$*, for all* $k \in [0..\lambda)$*; and (2)* $\|\rho\|^{\langle \mathbf{T}, \mathbf{T} \rangle} = \|\rho\|^{\mathbf{T}}$. □

Accordingly, any total HT-trace $\langle \mathbf{T}, \mathbf{T} \rangle$ can be seen as the LDL-trace $\mathbf{T}$. In fact, under total models, the satisfaction of dynamic operators $\langle \rho \rangle$ and $[\rho]$ in DHT collapses to that in LDL. Moreover, the first item implies that any DHT tautology is also an LDL tautology, so the former constitutes a weaker logic. To show that, in fact, DHT is *strictly* weaker, note that it does not satisfy some classical tautologies like the *excluded middle* $\varphi \vee \neg\varphi$, while LDL is a proper extension of classical logic. In fact, the addition of the axiom schema

$$\Box(a \vee \neg a) \quad \text{for each atom } a \in \mathcal{A} \text{ in the alphabet} \tag{EM}$$

forces total models and so, makes DHT collapse to LDL. Propositions 4 and 5 imply that φ is DHT_f satisfiable iff it is LDL_f satisfiable. Since the latter is a PSPACE-complete problem [7], the same applies to DHT_f satisfiability.

The next theorem shows that derived operators follow the expected definitions from HT and THT (and LTL).

Theorem 1. *Let* $\mathbf{M} = \langle \mathbf{H}, \mathbf{T} \rangle$ *be an* HT*-trace of length* λ *over alphabet* $\mathcal{A}$*. Given the respective definitions of derived operators, we get the following satisfaction conditions:*

1. $\mathbf{M}, k \models \varphi \wedge \psi$ *iff* $\mathbf{M}, k \models \varphi$ *and* $\mathbf{M}, k \models \psi$
2. $\mathbf{M}, k \models \varphi \vee \psi$ *iff* $\mathbf{M}, k \models \varphi$ *or* $\mathbf{M}, k \models \psi$
3. $\mathbf{M}, k \models \varphi \rightarrow \psi$ *iff* $\mathbf{M}', k \not\models \varphi$ *or* $\mathbf{M}', k \models \psi$*, for both* $\mathbf{M}' = \mathbf{M}$ *and* $\mathbf{M}' = \langle \mathbf{T}, \mathbf{T} \rangle$
4. $\mathbf{M}, k \models \neg\varphi$ *iff* $\langle \mathbf{T}, \mathbf{T} \rangle, k \not\models \varphi$
5. $\mathbf{M}, k \models \mathbb{F}$ *iff* $k + 1 = \lambda$
6. $\mathbf{M}, k \models \circ\varphi$ *iff* $k + 1 < \lambda$ *and* $\mathbf{M}, k{+}1 \models \varphi$
7. $\mathbf{M}, k \models \widehat{\circ}\varphi$ *iff* $k + 1 = \lambda$ *or* $\mathbf{M}, k{+}1 \models \varphi$
8. $\mathbf{M}, k \models \Diamond\varphi$ *iff* $\mathbf{M}, i \models \varphi$ *for some* $i \in [k..\lambda)$
9. $\mathbf{M}, k \models \Box\varphi$ *iff* $\mathbf{M}, i \models \varphi$ *for all* $i \in [k..\lambda)$
10. $\mathbf{M}, k \models \varphi \,\mathbb{U}\, \psi$ *iff for some* $j \in [k..\lambda)$*, we have* $\mathbf{M}, j \models \psi$ *and* $\mathbf{M}, i \models \varphi$ *for all* $i \in [k..j)$
11. $\mathbf{M}, k \models \varphi \,\mathbb{R}\, \psi$ *iff for all* $j \in [k..\lambda)$*, we have* $\mathbf{M}, j \models \psi$ *or* $\mathbf{M}, i \models \varphi$ *for some* $i \in [k..j)$
12. $\mathbf{M}, k \models \mathbf{I}$ *iff* $k = 0$
13. $\mathbf{M}, k \models \bullet\varphi$ *iff* $k > 0$ *and* $\mathbf{M}, k{-}1 \models \varphi$
14. $\mathbf{M}, k \models \widehat{\bullet}\varphi$ *iff* $k = 0$ *or* $\mathbf{M}, k{-}1 \models \varphi$
15. $\mathbf{M}, k \models \blacksquare\varphi$ *iff* $\mathbf{M}, i \models \varphi$ *for all* $i \in [0..k]$
16. $\mathbf{M}, k \models \blacklozenge\varphi$ *iff* $\mathbf{M}, i \models \varphi$ *for some* $i \in [0..k]$
17. $\mathbf{M}, k \models \varphi \,\mathbf{S}\, \psi$ *iff for some* $j \in [0..k]$*, we have* $\mathbf{M}, j \models \psi$ *and* $\mathbf{M}, i \models \varphi$ *for all* $i \in [j+1..k]$
18. $\mathbf{M}, k \models \varphi \,\mathbf{T}\, \psi$ *iff for all* $j \in [0..k]$*, we have* $\mathbf{M}, j \models \psi$ *or* $\mathbf{M}, i \models \varphi$ *for some* $i \in [j+1..k]$

as well as the relation:

19. $\|\phi\|^{\mathbf{M}} = \{(i, i+1) \mid \mathbf{M}, i \models \phi\}$ *for any propositional formula* ϕ. $\square$

An important observation above is that the satisfaction conditions for the Boolean operators amounts to standard HT while the interpretation of LTL operators (temporal formulas) subsume all the different previous versions of the *Temporal logic of Here and There* (THT), including the original definition for infinite traces [1], its extension to past operators [2], and its variant on finite traces [5].

Corollary 1. *Let φ be a temporal formula, $\mathbf{M}$ an HT-trace and $k \geq 0$. Then, $\mathbf{M}, k \models \varphi$ under THT satisfaction iff $\mathbf{M}, k \models \varphi$ under DHT satisfaction.* $\square$

Since our new definition also subsumes DHT for infinite traces [4] (when $\lambda = \omega$), we may classify all these previous approaches as follows. In analogy to [5], we consider several logics that are stronger than DHT and that can be obtained by the addition of axioms (or the corresponding restriction on sets of traces). For instance, we denote [4] as DHT_ω and define it as $\mathrm{DHT} + \{\neg\Diamond\mathbb{F}\}$, that is, DHT where we exclusively consider infinite HT-traces. The finite-trace version, we call DHT_f, corresponds to $\mathrm{DHT} + \{\Diamond\mathbb{F}\}$ instead. Linear Dynamic Logic for possibly infinite traces, LDL, can be obtained as $\mathrm{DHT} + \{(\mathrm{EM})\}$, that is, DHT with total HT-traces. Accordingly, we can define LDL_ω as $\mathrm{DHT}_\omega + \{(\mathrm{EM})\}$, i.e. infinite and total HT-traces, and obtain LDL_f as $\mathrm{DHT}_f + \{(\mathrm{EM})\}$, that is, LDL on finite traces [7]. Then, the variants THT_ω, THT_f, LTL_ω, LTL_f respectively refer to DHT_ω, DHT_f, LDL_ω, LDL_f on the restricted syntax of temporal formulas.

We now introduce non-monotonicity by selecting a particular set of traces that we call *temporal equilibrium models*. First, given an arbitrary set $\mathfrak{S}$ of HT-traces, we define the ones in equilibrium as follows.

Definition 2 (Temporal Equilibrium/Stable models). *Let $\mathfrak{S}$ be some set of HT-traces. A total HT-trace $\langle \mathbf{T}, \mathbf{T} \rangle \in \mathfrak{S}$ is an* equilibrium trace *of $\mathfrak{S}$ iff there is no other $\langle \mathbf{H}, \mathbf{T} \rangle \in \mathfrak{S}$ such that $\mathbf{H} < \mathbf{T}$. If this is the case, we also say that trace $\mathbf{T}$ is a* stable trace *of $\mathfrak{S}$. We further talk about* temporal equilibrium *or* temporal stable models *of a theory Γ when $\mathfrak{S} = \mathrm{DHT}(\Gamma)$, respectively.* $\square$

We write $\mathrm{DEL}(\Gamma, \lambda)$ and $\mathrm{DEL}(\Gamma)$ to stand for the temporal equilibrium models of $\mathrm{DHT}(\Gamma, \lambda)$ and $\mathrm{DHT}(\Gamma)$ respectively. Note that, due to Proposition 5, stable traces in $\mathrm{DEL}(\Gamma)$ are also LDL-models of Γ and, thus, DEL is stronger than LDL. Besides, as the ordering relation among traces is only defined for a fixed λ, it is easy to see:

Proposition 6. *The set of temporal equilibrium models of Γ can be partitioned by the trace length λ, that is, $\bigcup_{\lambda=0}^{\omega} \mathrm{DEL}(\Gamma, \lambda) = \mathrm{DEL}(\Gamma)$.* $\square$

(Linear) *Dynamic Equilibrium Logic* (DEL) is the non-monotonic logic induced by temporal equilibrium models of dynamic theories. We obtain the variants DEL_ω and DEL_f by applying the corresponding restriction to infinite or finite traces, respectively.

To illustrate non-monotonicity, take the formula:

$$[(\neg h)^*]\,(\neg h \rightarrow s) \tag{1}$$

whose reading is “keep sending an sos (s) while no help (h) is perceived.” Intuitively, $[(\neg h)^*]$ behaves as a conditional referring to any future state after $n \geq 0$ repetitions of

$(\neg h?;\top)$. Then, $\neg h \rightarrow s$ checks whether h fails one more time at $k = n$: if so, it makes s true again. Without additional information, this formula has a unique temporal stable model per each length λ satisfying $\Box(\neg h \wedge s)$, that is, h is never concluded, and so, we repeat s all over the trace. Suppose we add now the formula $\langle \top^5 \rangle\, h$, that is, h becomes true after five transitions. Then, there is a unique temporal stable model for each $\lambda > 5$ satisfying:

$$\langle (\neg h \wedge s)^5; h \wedge \neg s; (\neg h \wedge \neg s)^* \rangle\, \top$$

Clearly, $\Box(\neg h \wedge s)$ is not entailed any more (under temporal equilibrium models) showing that DEL is non-monotonic.

One important logical feature that emerges when dealing with a non-monotonic logic is the concept of *strong equivalence* [14]. Under a non-monotonic inference relation, the fact that two theories Γ_1 and Γ_2 yield the same consequences is too weak to consider that one can be "safely" replaced by the other, since the addition of new information Γ may make them behave in a different way. Instead, we normally define a stronger notion of equivalence, requiring that $\Gamma_1 \cup \Gamma$ and $\Gamma_2 \cup \Gamma$ have the same behavior, for any additional theory Γ (providing a context). An important property proved in [14] is that strong equivalence of propositional logic programs (and in fact, of arbitrary propositional theories) corresponds to regular equivalence in the monotonic logic of HT. This result reinforces the adequacy of the logic of HT as a monotonic basis for equilibrium logic and Answer Set Programming. Now, considering our setting, we can still prove that DHT plays a similar role with respect to DEL. Formally, we say that two dynamic theories Γ_1, Γ_2 are *strongly equivalent* if $\Gamma_1 \cup \Gamma$ and $\Gamma_2 \cup \Gamma$ have the same temporal equilibrium models, for any additional LDL theory Γ. Then, we get the following result, by a direct application of the proof obtained for (converse-free) DHT_ω (Theorem 2 in [4]) to the general case with converse operator and arbitrary length $\lambda \in \mathbb{N} \cup \{\omega\}$:

Theorem 2. *Dynamic theories Γ_1 and Γ_2 are strongly equivalent iff $\Gamma_1 \equiv \Gamma_2$ in* DHT.

This result shows that DHT-equivalence precisely captures the property of strong equivalence of dynamic theories. Thus, it is worth commenting some possible ways of deriving DHT equivalences. We already know that any DHT equivalence must also hold in LDL while, in general, the opposite does not hold, as with $[\rho]\, q \equiv \neg\langle\rho\rangle\, \neg q$. Still, some LDL equivalences are preserved in DHT, like the following unfolding properties.

Proposition 7. *The following equivalences hold in* DHT.

$$\begin{aligned} \langle \rho + \rho' \rangle\, \varphi &\equiv \langle \rho \rangle\, \varphi \vee \langle \rho' \rangle\, \varphi & \qquad [\rho + \rho']\, \varphi &\equiv [\rho]\, \varphi \wedge [\rho']\, \varphi \\ \langle \rho\, ;\, \rho' \rangle\, \varphi &\equiv \langle \rho \rangle\, \langle \rho' \rangle\, \varphi & [\rho\, ;\, \rho']\, \varphi &\equiv [\rho]\, [\rho']\, \varphi \\ \langle \rho^* \rangle\, \varphi &\equiv \varphi \vee \langle \rho \rangle\, \langle \rho^* \rangle\, \varphi & [\rho^*]\, \varphi &\equiv \varphi \wedge [\rho]\, [\rho^*]\, \varphi \end{aligned}$$

In Proposition 2 in [4], we proved that (converse-free) LDL_ω equivalences for unconditional formulas can also be guaranteed in DHT_ω. We extend below the same result for DHT and LDL with converse operator and traces of arbitrary length.

Proposition 8. *For unconditional formulas φ and ψ, $\varphi \equiv \psi$ in* LDL *iff $\varphi \equiv \psi$ in* DHT.

This result suffices to prove the three leftmost equivalences above by resorting to LDL, but cannot be applied for proving the right ones, as they are conditional — they contain arbitrary path expressions inside $[\cdot]$. An interesting fragment are temporal formulas without $\to$ or $\neg$. They are unconditional, since the definition of temporal operators only use $[\cdot]$ for $\Box\varphi = [\top^*]\,\varphi$ and its dual $\blacksquare\varphi = [\top^{*-}]\,\varphi$, and these formulas do not use atoms in the path expressions. As a consequence of Proposition 8, the DHT equivalence for temporal formulas without implications or negations can be directly checked in LTL.

Given any dynamic formula φ, we define φ^- as the result of replacing in φ each (maximal) path expression ρ by ρ^-. For instance, given $\varphi = [p;q]\,\langle r^-\rangle\, s$ we get $\varphi = [(p;q)^-]\,\langle r^{--}\rangle\, s$. Notice that the effect of this transformation on temporal operators is just switching their future/past versions. As an example:

$$(\Diamond\widehat{\bullet}p)^- = (\langle\top^*\rangle\,[\top^-]\,p)^- = \langle\top^{*-}\rangle\,[\top^{--}]\,p = \blacklozenge\widehat{\circ}p$$

Lemma 1. *There exists a mapping ϱ on finite* HT*-traces of a fixed length $\lambda = n \in \mathbb{N}$ such that, $\mathbf{M}, k \models \varphi$ iff $\varrho(\mathbf{M}), n-k \models \varphi^-$, for any $k \in [0..\lambda)$, any dynamic formula φ and any* HT*-trace $\mathbf{M}$ of length $\lambda = n$.*

Theorem 3 (Temporal duality theorem). *A dynamic formula φ is a DHT_f tautology iff φ^- is a DHT_f tautology.*

This property does not hold for infinite traces, where $\neg\Diamond\mathbb{F}$ is valid but its dual, $\neg\blacklozenge\mathbf{I}$, is false in all traces (we can always reach the initial situation at some point in the past).

3 Reducing converse-free DEL_f to propositional ASP

In this section, we show that converse-free DEL_f can be reduced to propositional theories (under stable models semantics) by using indexed atoms. Given a set $\mathcal{A}$ of atoms and $\lambda \in \mathbb{N}$, we define $\mathcal{A}^\lambda \overset{def}{=} \{a_i \mid i \in [0..\lambda) \text{ and } a \in \mathcal{A}\}$. We define the translation of a converse-free dynamic formula φ at $i \in [0..\lambda)$, in symbols $\big(\varphi\big)_i$, as follows:

$$\big(\bot\big)_i \overset{def}{=} \bot \qquad \big(\top\big)_i \overset{def}{=} \top \qquad \big(p\big)_i \overset{def}{=} p_i \quad \text{for each } p \in \mathcal{A}$$

$$\big(\langle\varphi?\rangle\,\psi\big)_i \overset{def}{=} \big(\varphi\big)_i \wedge \big(\psi\big)_i \qquad\qquad \big([\varphi?]\,\psi\big)_i \overset{def}{=} \big(\varphi\big)_i \to \big(\psi\big)_i$$

$$\big(\langle\top\rangle\,\varphi\big)_i \overset{def}{=} \begin{cases} \big(\varphi\big)_{i+1} & \text{if } i+1 < \lambda \\ \bot & \text{if } i+1 = \lambda \end{cases} \qquad \big([\top]\,\varphi\big)_i \overset{def}{=} \begin{cases} \big(\varphi\big)_{i+1} & \text{if } i+1 < \lambda \\ \top & \text{if } i+1 = \lambda \end{cases}$$

and, for any other formula α starting with $\langle\cdot\rangle$ or $[\cdot]$, we apply the equivalences $\alpha \equiv \beta$ in Proposition 7 to unfold $\big(\alpha\big)_i$ into $\big(\beta\big)_i$, further assuming $\big(\varphi \otimes \psi\big)_i = \big(\varphi\big)_i \otimes \big(\psi\big)_i$ for $\otimes \in \{\wedge, \vee\}$. As an example, consider the formula $[p^*]\,q$ and assume that $\lambda = 3$:

$$\begin{aligned}
\big([p^*]\,q\big)_0 &= \big(q\big)_0 \wedge \big([p]\,[p^*]\,q\big)_0 \\
&= q_0 \wedge \big([p?]\,[\top]\,[p^*]\,q\big)_0 \\
&= q_0 \wedge (p_0 \to \big([p^*]\,q\big)_1) && \text{then, repeating the pattern} \\
&= q_0 \wedge (p_0 \to (q_1 \wedge (p_1 \to \big([p^*]\,q\big)_2))) && \text{that is HT-equivalent to} \\
&= q_0 \wedge (p_0 \to q_1) \wedge (p_0 \wedge p_1 \to \big([p^*]\,q\big)_2) \\
&= q_0 \wedge (p_0 \to q_1) \wedge (p_0 \wedge p_1 \to (p_2 \wedge \underbrace{\big([p]\,[p^*]\,q\big)_2}_{\top})) \\
&= q_0 \wedge (p_0 \to q_1) \wedge (p_0 \wedge p_1 \to p_2)
\end{aligned}$$

It is easy to see that, applying the same pattern for $\big((1)\big)_0$ and $\lambda = 3$, we get:

$$\begin{aligned}
&(\neg h_0 \to s_0) \wedge (\neg h_0 \to (\neg h_1 \to s_1)) \wedge (\neg h_0 \wedge \neg h_1 \to (\neg h_2 \to s_2)) \\
= &(\neg h_0 \to s_0) \wedge (\neg h_0 \wedge \neg h_1 \to s_1) \wedge (\neg h_0 \wedge \neg h_1 \wedge \neg h_2 \to s_2)
\end{aligned}$$

Theorem 4 (Partial correctness). *Let $\big(\alpha\big)_i$ terminate for formula α and $i \in [0..\lambda)$ with $\lambda \in \mathbb{N}$. For any finite* HT*-trace $\mathbf{M} = \langle \mathbf{H}, \mathbf{T} \rangle$ of length λ, and its (one-to-one) corresponding* HT*-interpretation $M = \langle \{a_i \mid a \in H_i\}, \{a_i \mid a \in T_i\} \rangle$ on $\mathcal{A}^\lambda$, we have $\mathbf{M}, i \models \alpha$ in* DHT *iff $M \models \big(\alpha\big)_i$ in* HT.

As stated above, the previous result only guarantees a *partial correctness* for the recursive translation $\big(\varphi\big)_i$ — to get *total correctness* we further need to guarantee termination, and this does not hold in the general case. To see why, just consider the formula $[\top?^*]\, q$ (being equivalent to q) whose translation at i yields

$$\big([\top?^*]\, q\big)_i = q_i \wedge \big([\top?]\,[\top?^*]\, q\big)_i = q_i \wedge (\top \to \big([\top?^*]\, q\big)_i) = q_i \wedge \big([\top?^*]\, q\big)_i$$

and generates an infinite sequence of calls to $\big([\top?^*]\, q\big)_i$. This problem arises because the starred expression, $\top?$, leaves situation i unaltered, something that does not happen with $p^* = (p?;\top)^*$ used before, as it generated incremental jumps $i+1 > i$ and a sequence of calls $\big([p^*]\, q\big)_0$, $\big([p^*]\, q\big)_1$, $\big([p^*]\, q\big)_2$ progressing towards $i = \lambda - 1$. We show next that any converse-free path expression ρ^* can be equivalently reformulated in such a way that its translation proceeds in a strictly incremental way, guaranteeing termination. We begin by defining the following types of path expressions: a *(sequential) component* θ, a *sequence* σ and a *normalized disjunction* δ are defined by the grammar rules:

$$\theta ::= \top \mid \varphi? \mid \delta^* \qquad \sigma ::= \theta \mid \sigma_1;\sigma_2 \qquad \delta ::= \theta \mid \delta_1 + \delta_2$$

Given that addition satisfies distributivity with respect to sequence, viz.

$$(\rho_1 + \rho_2);\rho_3 \equiv (\rho_1;\rho_2) + (\rho_2;\rho_3) \qquad \rho_1;(\rho_2 + \rho_3) \equiv (\rho_1;\rho_2) + (\rho_1;\rho_3),$$

it is easy to obtain the following result.

Proposition 9 (Disjunctive Normal Form). *Any arbitrary path expression ρ can be equivalently reformulated as a normalized disjunction δ.*

As an example, to normalize the expression $(a^* + b);(c?;d + e?)^*$ we can proceed, for instance, by reducing the inner expression $(c?;d + e?)$ to $(c?;d) + (c?;e?)$ and then go on applying distributivity outside:

$$\begin{aligned}
(a^* + b);(c?;d + e?)^* &= (a^* + b);((c?;d) + (c?;e?))^* \\
&= (a^*;((c?;d) + (c?;e?))^*) + (b;((c?;d) + (c?;e?))^*)
\end{aligned}$$

The last expression is already in normal form. We say that a sequence $\sigma = \theta_1;\ldots;\theta_n$ is *incremental* if $\theta_i = \top$ for some $i = 1,\ldots,n$. A normalized disjunction $\delta = \sigma_1 + \cdots + \sigma_m$ is *incremental* if σ_i is incremental for every $i = 1,\ldots,m$.

Proposition 10. *Let δ be an incremental, normalized disjunction and $\mathbf{M}$ an HT-trace. Then, $(i,j) \in \|\delta\|^{\mathbf{M}}$ implies $j > i$.*

In other words, incremental disjunctions always shift the time point strictly forward. Obviously, not any normalized disjunction δ is incremental, but this is not a problem as long as it is not combined with the star operator. We say that a normalized disjunction δ is *star-incremental* if all the sub-expressions $(\delta')^*$ of δ satisfy that δ' is incremental. The key point for guaranteeing termination is that we can transform any arbitrary path expression into a star-incremental, normalized disjunction.

Proposition 11. *For any expression ρ and formulas $\varphi_1, \ldots, \varphi_n$, we have that $(\rho + (\varphi_1?; \ldots; \varphi_n?))^* = (\rho)^*$ and $(\varphi_1?; \ldots; \varphi_n?)^* = \top?$.*

In other words, we can remove test-only sequences from any iterated disjunction. As an example $(\,(c?; d) \,+\, (c?; e?)\,)^*$ amounts to $(c?; d)^*$. Similarly, if we only have tests $(a?; b?)^*$ the whole expression can be just replaced by $\top?$.

Now, to transform any normalized disjunction to become star-incremental, we can proceed in a bottom-up manner, as described in the proof of the following proposition, included below to illustrate the process.

Proposition 12. *Any converse-free path expression can be transformed into an equivalent star-incremental, normalized disjunction.*

Proof. By Proposition 9, we can assume that we start from a normalized disjunction. Then, we begin with all the sub-expressions δ^* where δ is star-free, and so, trivially star-incremental. To make δ^* star-incremental too, it suffices with removing its test-only sequences, applying Proposition 11. Then, we proceed with δ^* where δ is not star-free, but is already star-incremental by application of previous steps. Any non-incremental sequence in δ is a combination of tests and starred expressions. Suppose we take some non-incremental sequence of δ of the form $\sigma; \rho^*; \sigma'$. Note that, if σ_1 or σ_2 are not present, we can assume they correspond to $\top?$. Then, we can apply the unfolding:

$$\begin{aligned} \sigma; \rho^*; \sigma' &= \sigma; (\top? + \rho; \rho^*); \sigma' \\ &= (\sigma; \top?; \sigma') + (\sigma; \rho; \rho^*; \sigma') \\ &= (\sigma; \sigma') + (\sigma; \rho; \rho^*; \sigma') \end{aligned}$$

where ρ^* does not occur in the first sequence, whereas in the second, we can apply distributivity on all sequences from the first occurrence of ρ. Since δ is star-incremental, ρ is incremental and so, all the sequences obtained in that way are incremental too. We would then proceed in the same way with the next starred-expression in $(\sigma; \sigma')$. The final result, δ', is equivalent to δ but contains incremental sequences or test-only expressions. But in $(\delta')^*$ we can further remove the test-only expressions (Property 11) and we eventually get a star-incremental expression. $\square$

As an example, given $(a?; (b+c)^*; d?; e^*)^*$, we can unfold $(b+c)^*$ into

$$(a?; \underline{(b+c)^*}; d?; e^*)^* = (\,(a?; d?; e^*) \,+\, (a?; (b+c); (b+c)^*; d?; e^*))^*$$

and unfold again the first sequence into:

$$= (\,(a?; d?) \,+\, (a?; d?; e; e^*) \,+\, (a?; (b+c); (b+c)^*; d?; e^*))^*$$

By Proposition 11, the test-only sequence $(a?; d?)$ can be removed

$$= (\,(a?; d?; e; e^*) \;+\; (a?; (b+c); (b+c)^*; d?; e^*))^*$$

and, now, applying distributivity on $(b+c)$, we get

$$\begin{aligned}
&= (\,(a?; d?; e; e^*) \;+\; (a?; b; (b+c)^*; d?; e^*) \;+\; (a?; c; (b+c)^*; d?; e^*))^* \\
&= (\,(a?; d?; e?; \top; e^*) \;+\; (a?; b?; \top; (b+c)^*; d?; e^*) \;+\; (a?; c?; \top; (b+c)^*; d?; e^*))^*
\end{aligned}$$

and all remaining sequences are incremental.

Theorem 5. *Let φ be a formula where all its path expressions are star-incremental, normalized disjunctions, and let $i \in [0..\lambda)$ with $\lambda \in \mathbb{N}$, $\lambda > 0$. Then, $\big(\varphi\big)_i$ terminates.*

Corollary 2. *Given a fixed length $\lambda \in \mathbb{N}$, any converse-free dynamic theory can be reduced to a propositional theory with a one-to-one correspondence among the respective* HT*-traces (of length λ) and* HT*-models.*

Given that any propositional theory can be translated into an HT-equivalent disjunctive logic program (cf. [6]), we get the following result.

Corollary 3. *Given a fixed length $\lambda \in \mathbb{N}$, any converse-free dynamic theory can be reduced to a disjunctive logic program with a one-to-one correspondence among the respective* HT*-traces (of length λ) and* HT*-models.*

4 Discussion and conclusions

As we have seen, our current definition of Dynamic Equilibrium Logic (DEL), covers the previous modal variants of Equilibrium Logic for dealing with time, including the original Temporal Equilibrium Logic (TEL) [1], its extension to past operators [2] and its variant on finite traces [5], but also generalizes the first definition of DEL in [4] by possibly allowing for finite traces and a converse operator. The recent introduction of Dynamic Logic operators in modal Equilibrium Logic and the use of finite traces have been obviously motivated by [7], that previously presented LTL and LDL on finite traces. DEL can be seen as a non-monotonic extension that allows for capturing temporal stable models of LDL theories. As happens in the non-temporal case, when we add the excluded middle axiom, DEL and TEL respectively collapse to the monotonic versions LDL and LTL. A different approach for extending ASP with linear-time and dynamic operators was studied in [8], for a rule-based syntax, and later generalized in [3] for arbitrary dynamic logic theories. The main difference with respect to DEL is that [8] starts from the linear version of DL in [10] and keeps separate alphabets for atomic actions and propositions. Still, as shown in [4], both [8] and [3] can be encoded in DEL_ω. The approaches in [18, 17] give encodings of GOLOG-like control in ASP planning by enforcing that traces are compatible with a given path expression without any logical underpinnings.

Apart from the general definition of DEL and its relation to other formalisms, a second contribution of the paper is the translation of any converse-free arbitrary DEL_f

theory into a propositional logic program. This translation has proved to be non-trivial: it is based on unfolding path expressions, something potentially equivalent to the execution of a sequential program. Termination was guaranteed by a previous preprocessing of path expressions. Future work includes the implementation of this translation for the converse-free fragment of the language together with the extension to other fragments involving the converse operator.

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