Temporal Answer Set Programming on Finite Traces

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Outline

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Answer Set Programming (ASP)

- What is ASP?
  ASP is an approach for declarative problem solving

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  Examples Sudoku, Configuration, Diagnosis, Music composition, Planning, System design, Time tabling, etc.

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Potassco

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  - Debian, Ubuntu: Linux package configuration
  - Exeura: Call routing
  - Fcc: Radio frequency auction
  - Gioia Tauro: Workforce management
  - Nasa: Decision support for Space Shuttle
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> Over 13 months in 2016–17 the US Federal Communications Commission conducted an "incentive auction" to repurpose radio spectrum from broadcast television to wireless internet. In the end, the auction yielded $19.8 billion, $10.05 billion of which was paid to 175 broadcasters for voluntarily relinquishing their licenses across 14 UHF channels. Stations that continued broadcasting were assigned potentially new channels to fit as densely as possible into the channels that remained. The government netted more than $7 billion (used to pay down the national debt) after covering costs. A crucial element of the auction design was the construction of a solver, dubbed SATFC, that determined whether sets of stations could be "repacked" in this way; it needed to run every time a station was given a price quote. This

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- Anything not so good for ASP?
  - Number crunching

Some biased moments in time

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  - Databases Closed world assumption
  - Logic programming Negation as failure
  - Non-monotonic reasoning
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    - Extensions of first-order logic
    - Modalities, fix-points, second-order logic

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    Well-founded and stable models semantics
  - ASP solving
    "Stable models = Well-founded semantics + Branch"

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    "Stable models = Well-founded semantics + Branch"
    - Modeling — Grounding — Solving
    - Icebreakers: `lparse` and `smodels`

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  - Constructive logics Equilibrium Logic

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    - Roots: Logic of Here-and-There (Heyting'30), G3 (Gödel'32)

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Robotic intra-logistic

- Robotics systems for logistics and warehouse automation based on hundreds of
  - mobile robots
  - movable shelves
- Main tasks: order fulfillment, i.e.
  - routing
  - order picking
  - replenishment
- Many competing industry solutions:
  - Amazon, Dematic, Genzebach, Gray Orange, Swisslog

Robotic intra-logistic in ASP
routing

```
time(1..horizon).

direction((X,Y)) :- X=-1..1, Y=-1..1, |X+Y|=1.
nextto((X,Y),(X',Y'),(X+X',Y+Y')) :- position((X,Y)), direction((X',Y')), position((X+X',Y+Y')).

 { move(R,D,T) : direction(D) } 1 :- isRobot(R), time(T).

position(R,C,T) :- move(R,D,T), position(R,C',T-1),     nextto(C',D,C).
                :- move(R,D,T), position(R,C ,T-1), not nextto(C ,D,_).

position(R,C,T) :- position(R,C,T-1), not move(R,_,T), isRobot(R), time(T).

moveto(C',C,T) :- nextto(C',D,C), position(R,C',T-1), move(R,D,T).
 :- moveto(C',C,T), moveto(C,C',T).

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Robotic intra-logistic in ASP
routing to shelves

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processed(O,A) :- ordered(O,A), shelved(S,A), position(S,C,0), position(R,C,horizon), isRobot(R).

processed(O) :- isOrder(O), processed(O,A) : ordered(O,A).

:- not processed(O), isOrder(O).
```

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Robotic intra-logistic in ASP

routing + transport + delivery

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 {    move(R,D,T) : direction(D) ;
    pickup(R,S,T) : isShelf(S)   ;
   putdown(R,S,T) : isShelf(S)   } 1 :- isRobot(R), time(T).

waits(R,T) :- not pickup(R,_,T), not putdown(R,_,T), not move(R,_,T), isRobot(R), time(T).

position(R,C,T) :- move(R,D,T),    position(R,C',T-1),     nextto(C',D,C).
                :- move(R,D,T),    position(R,C ,T-1), not nextto(C, D,_).

 carries(R,S,T) :- pickup(R,S,T),  position(R,C,T-1), position(S,C ,T-1).
                :- pickup(R,S,T),   carries(R,_,T-1).
                :- pickup(R,S,T),   carries(_,S,T-1).
                :- pickup(R,S,T),  position(R,C,T-1), position(S,C',T-1), C != C'.

                :- putdown(R,S,T), not carries(R,S,T-1).

serves(R,S,P,T) :- position(R,C,T), carries(R,S,T),  position(P,C), isStation(P).

position(R,C,T) :- position(R,C,T-1), not    move(R,_,T), isRobot(R), time(T).
 carries(R,S,T) :-  carries(R,S,T-1), not putdown(R,_,T),             time(T).

position(S,C,T) :- position(R,C,T  ),     carries(R,S,T).
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moveto(C',C,T) :- nextto(C',D,C), position(R,C',T-1), move(R,D,T).
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    - complexity beyond *LTL*
    - infinite traces
  - Action languages
    - static and dynamic laws
    - same complexity as ASP
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  $\sim$ LTL_f by G. De Giacomo and M. Vardi (2013)

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Potassco

Regular formulas

- Formulas

$$\varphi ::= a \mid \bot \mid \varphi_1 \otimes \varphi_2$$

  where
  - a is an atom
  - $\otimes$ is a binary Boolean connective among $\rightarrow, \wedge, \vee$

- Defined connectives
  - $\top = \neg\bot$
  - $\neg\varphi = \varphi \rightarrow \bot$
  - $\varphi \leftrightarrow \psi = (\varphi \rightarrow \psi) \wedge (\psi \rightarrow \varphi)$

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* in the logic of Here-and-There (Heyting'32; Gödel'32)

Temporal formulas

- Temporal operators

past	$\bullet$	for *previous*	*future*	$\circ$	for *next*
	$\mathbf{S}$	for *since*		$\mathbb{U}$	for *until*
	$\mathbf{T}$	for *trigger*		$\mathbb{R}$	for *release*

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$$\varphi ::= a \mid \bot \mid \varphi_1 \otimes \varphi_2 \mid \bullet\varphi \mid \varphi_1 \,\mathbf{S}\, \varphi_2 \mid \varphi_1 \,\mathbf{T}\, \varphi_2 \mid \circ\varphi \mid \varphi_1 \,\mathbb{U}\, \varphi_2 \mid \varphi_1 \,\mathbb{R}\, \varphi_2$$

- Defined operators

$\blacksquare\varphi = \bot \,\mathbf{T}\, \varphi$	*always before*	$\Box\varphi = \bot \,\mathbb{R}\, \varphi$	*always afterward*
$\blacklozenge\varphi = \top \,\mathbf{S}\, \varphi$	*eventually before*	$\Diamond\varphi = \top \,\mathbb{U}\, \varphi$	*eventually afterward*
$\mathbf{I} = \neg\bullet\top$	*initial*	$\mathbb{F} = \neg\circ\top$	*final*
$\widehat{\bullet}\varphi = \bullet\varphi \vee \mathbf{I}$	*weak previous*	$\widehat{\circ}\varphi = \circ\varphi \vee \mathbb{F}$	*weak next*

Examples

- *"If we shoot twice with a gun that was never loaded, it will eventually fail."*

$$\Box(\mathit{shoot} \wedge \bullet\blacklozenge\mathit{shoot} \wedge \blacksquare\mathit{unloaded} \rightarrow \Diamond\mathit{fail})$$

- *"Why does shooting a loaded gun fail in unloading it?"*

$$\Box(\mathbb{F} \rightarrow \neg\neg(\mathit{shoot} \wedge \bullet\mathit{loaded} \wedge \mathit{loaded}))$$

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- *HT*-Model A pair $\langle H, T \rangle$ of set of atoms st $H \subseteq T \subseteq \mathcal{A}$

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  - $\mathbf{H} \leq \mathbf{H}'$ if $H_i \subseteq H'_i$ for $i = 0..\lambda$

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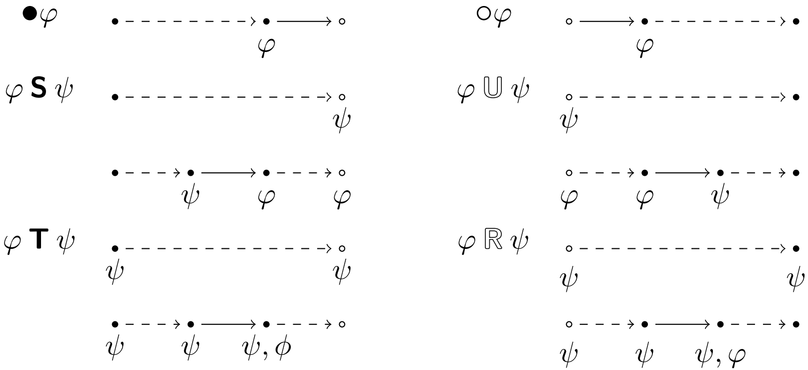
- Notation We abbreviate $\langle H_i, T_i \rangle_{i=0}^{\lambda}$ by $\langle \mathbf{H}, \mathbf{T} \rangle$
- Note $\mathbf{H} \leq \mathbf{T}$

Satisfaction of regular formulas

An *HT*-trace $\langle \mathbf{H}, \mathbf{T} \rangle$ of length λ over alphabet $\mathcal{A}$ *satisfies* a temporal formula φ at time point $k = 0..\lambda$, $k \neq \omega$, written $\langle \mathbf{H}, \mathbf{T} \rangle, k \models \varphi$, if the following conditions hold:

1. $\langle \mathbf{H}, \mathbf{T} \rangle, k \not\models \bot$
2. $\langle \mathbf{H}, \mathbf{T} \rangle, k \models a$ iff $a \in H_k$, for any atom $a \in \mathcal{A}$
3. $\langle \mathbf{H}, \mathbf{T} \rangle, k \models \varphi \wedge \psi$ iff $\langle \mathbf{H}, \mathbf{T} \rangle, k \models \varphi$ and $\langle \mathbf{H}, \mathbf{T} \rangle, k \models \psi$
4. $\langle \mathbf{H}, \mathbf{T} \rangle, k \models \varphi \vee \psi$ iff $\langle \mathbf{H}, \mathbf{T} \rangle, k \models \varphi$ or $\langle \mathbf{H}, \mathbf{T} \rangle, k \models \psi$
5. $\langle \mathbf{H}, \mathbf{T} \rangle, k \models \varphi \rightarrow \psi$ iff $\langle \mathbf{H}', \mathbf{T} \rangle, k \not\models \varphi$ or $\langle \mathbf{H}', \mathbf{T} \rangle, k \models \psi$,
   for all $\mathbf{H}' \in \{\mathbf{H}, \mathbf{T}\}$

Satisfaction of temporal formulas



Satisfaction of temporal formulas

6. $\langle \mathbf{H}, \mathbf{T} \rangle, k \models \bullet\varphi$ iff $k > 0$ and $\langle \mathbf{H}, \mathbf{T} \rangle, k{-}1 \models \varphi$
7. $\langle \mathbf{H}, \mathbf{T} \rangle, k \models \varphi \, \mathbf{S} \, \psi$ iff for some $j = 0..k$, we have $\langle \mathbf{H}, \mathbf{T} \rangle, j \models \psi$ and $\langle \mathbf{H}, \mathbf{T} \rangle, i \models \varphi$ for all $i = j{+}1..k$
8. $\langle \mathbf{H}, \mathbf{T} \rangle, k \models \varphi \, \mathbf{T} \, \psi$ iff for all $j = 0..k$, we have $\langle \mathbf{H}, \mathbf{T} \rangle, j \models \psi$ or $\langle \mathbf{H}, \mathbf{T} \rangle, i \models \varphi$ for some $i = j{+}1..k$

Satisfaction of temporal formulas

6 $\langle \mathbf{H}, \mathbf{T}\rangle, k \models \bullet\varphi$ iff $k > 0$ and $\langle \mathbf{H}, \mathbf{T}\rangle, k{-}1 \models \varphi$

7 $\langle \mathbf{H}, \mathbf{T}\rangle, k \models \varphi \,\mathbf{S}\, \psi$ iff for some $j = 0..k$, we have $\langle \mathbf{H}, \mathbf{T}\rangle, j \models \psi$ and $\langle \mathbf{H}, \mathbf{T}\rangle, i \models \varphi$ for all $i = j{+}1..k$

8 $\langle \mathbf{H}, \mathbf{T}\rangle, k \models \varphi \,\mathbf{T}\, \psi$ iff for all $j = 0..k$, we have $\langle \mathbf{H}, \mathbf{T}\rangle, j \models \psi$ or $\langle \mathbf{H}, \mathbf{T}\rangle, i \models \varphi$ for some $i = j{+}1..k$

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Satisfaction of temporal formulas

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Satisfaction of (defined) temporal formulas

12 $\langle \mathbf{H}, \mathbf{T} \rangle, k \models \top$

13 $\langle \mathbf{H}, \mathbf{T} \rangle, k \models \blacksquare\varphi$ iff $\langle \mathbf{H}, \mathbf{T} \rangle, i \models \varphi$ for all $i = 0..k$

14 $\langle \mathbf{H}, \mathbf{T} \rangle, k \models \blacklozenge\varphi$ iff $\langle \mathbf{H}, \mathbf{T} \rangle, i \models \varphi$ for some $i = 0..k$

15 $\langle \mathbf{H}, \mathbf{T} \rangle, k \models \mathbf{I}$ iff $k = 0$

16 $\langle \mathbf{H}, \mathbf{T} \rangle, k \models \widehat{\bullet}\varphi$ iff $k = 0$ or $\langle \mathbf{H}, \mathbf{T} \rangle, k{-}1 \models \varphi$

17 $\langle \mathbf{H}, \mathbf{T} \rangle, k \models \Box\varphi$ iff $\langle \mathbf{H}, \mathbf{T} \rangle, i \models \varphi$ for any $i = k..\lambda$

18 $\langle \mathbf{H}, \mathbf{T} \rangle, k \models \Diamond\varphi$ iff $\langle \mathbf{H}, \mathbf{T} \rangle, i \models \varphi$ for some $i = k..\lambda$

19 $\langle \mathbf{H}, \mathbf{T} \rangle, k \models \mathbb{F}$ iff $k = \lambda$

20 $\langle \mathbf{H}, \mathbf{T} \rangle, k \models \widehat{\circ}\varphi$ iff $k = \lambda$ or $\langle \mathbf{H}, \mathbf{T} \rangle, k{+}1 \models \varphi$

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- Note All variants of THT are monotonic !

Temporal equilibrium logic (*TEL*)

- A total *HT*-trace $\langle \mathbf{T}, \mathbf{T} \rangle$ is an equilibrium model of a temporal formula φ, if

  1. $\langle \mathbf{T}, \mathbf{T} \rangle, 0 \models \varphi$,
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  - $\Box \Diamond a$ yields
    - no model in TEL_ω but $(\emptyset^* \{a\})$ in TEL_f

Outline

Normalform

- Temporal literals $\{a, \neg a, \bullet a, \neg\bullet a \mid a \in \mathcal{A}\}$

- Temporal rules
  - initial rule $B \to A$
  - dynamic rule $\widehat{\circ}\,\square(B \to A)$
  - final rule $\square(\,\mathbb{F} \to (B \to A)\,)$

  where $B = b_1 \wedge \cdots \wedge b_n$ and $A = a_1 \vee \cdots \vee a_m$
  and b_i and a_j are temporal literals for dynamic rules,
  and regular literals for initial and final rules

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- Temporal logic program is a set of temporal rules
- Theorem Every temporal formula φ can be converted into a temporal logic program THT_f-equivalent to φ

Example

$$\widehat{\circ}\,\square(\bullet loaded \wedge \neg unloaded \rightarrow loaded)$$

$$\widehat{\circ}\,\square(shoot \wedge \bullet loaded \wedge loaded \rightarrow goal)$$

$$\square(\mathbb{F} \rightarrow (\neg goal \rightarrow \bot))$$

Bounded translation

- Temporal literals at time point k

$$\tau_k(a) = a_k \qquad \tau_k(\neg a) = \neg a_k$$
$$\tau_k(\bullet a) = a_{k-1} \qquad \tau_k(\neg \bullet a) = \neg a_{k-1}$$

- Temporal rules focusing on $B \to A$ at time point k

$$\tau_k(r) = \tau_k(a_1) \vee \cdots \vee \tau_k(a_m) \leftarrow \tau_k(b_1) \wedge \cdots \wedge \tau_k(b_n)$$

- Temporal logic program P bounded by finite length λ

$$\begin{aligned}\tau_\lambda(P) = \quad & \{\tau_0(r) \mid r \in I(P)\,\} \\ & \cup \{\tau_k(r) \mid r \in D(P), k = 1..\lambda\,\} \\ & \cup \{\tau_\lambda(r) \mid r \in F(P)\,\}\end{aligned}$$

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- Method module theory accounting for composition of logic programs

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$$\tau^*_k(r) = \tau_k(a_1) \vee \cdots \vee \tau_k(a_m) \leftarrow \tau_k(b_1) \wedge \cdots \wedge \tau_k(b_n) \wedge \neg q_{k+1}$$

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- *tel*
  - is a preprocessor
  - implements the bounded translation
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- Example

$$\{ \rightarrow a, \quad \widehat{\circ}\,\square(\bullet a \rightarrow b), \quad \square(\mathbb{F} \rightarrow (\neg b \rightarrow \bot)) \}$$

is represented as

```
a.
#next^ #always+ ( (#previous a) -> b).
#always+ ( #final -> (~ b -> #false)).
```

telingo

- *telingo*
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  - implements the incremental translation
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  - $\bullet p(a)$ and $\circ q(b)$ can be expressed by `'p(a)` and `q'(b)`

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- Primes allow for expressing (iterated) next and previous operators
  - $\bullet p(a)$ and $\circ q(b)$ can be expressed by `'p(a)` and `q'(b)`
- Example *"A robot cannot lift a box unless its capacity exceeds the box's weight plus that of all held objects"*

```
:- lift(R,B), robot(R), box(B,W),
   #sum {    C : capacity(R,C);
          -V,O : 'holding(R,O,V) } < W.
```

telingo's temporal logic programs

- initial rule $B \rightarrow A$
- dynamic rule $\widehat{\circ}\,\Box(B \rightarrow A)$
- final rule $\Box(\,\mathbb{F} \rightarrow (B \rightarrow A)\,)$

telingo's temporal logic programs

- initial rule $B \to A$
- dynamic rule $\widehat{\circ}\,\square(B \to A)$
- final rule $\square(\,\mathbb{F} \to (B \to A)\,)$
- always rule $\square(B \to A)$

telingo's temporal logic programs

- initial rule $B \rightarrow A$ `#program initial.`
- dynamic rule $\widehat{\circ}\,\square(B \rightarrow A)$ `#program dynamic.`
- final rule $\square(\,\mathbb{F} \rightarrow (B \rightarrow A)\,)$ `#program final.`
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- Example $\{\; \rightarrow a,\; \widehat{\circ}\,\square(\bullet a \rightarrow b),\; \square(\mathbb{F} \rightarrow (\neg b \rightarrow \bot))\;\}$

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- always rule $\square(B \to A)$ `#program always.`

- Example $\{ \to a,\ \widehat{\circ}\,\square(\bullet a \to b),\ \square(\mathbb{F} \to (\neg b \to \bot)) \}$
  can alternatively be represented as

```
#program initial.
a.

#program dynamic.
b :- 'a.

#program final.
  :- not b.
```

```
#program always.
a :- &initial.


b :- 'a.


  :- not b, &final.
```

telingo's temporal formulas

- `&initial` $\mathbf{I}$
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- `&initial` $\mathbf{I}$
- `&final` $\mathbb{F}$
- `&tel { `φ` }` for temporal formula φ
  - Temporal operators

past	$\bullet$	*previous*	*future*	$\circ$	*next*
	$\mathbf{S}$	*since*		$\mathbb{U}$	*until*
	$\mathbf{T}$	*trigger*		$\mathbb{R}$	*release*
	$\blacklozenge$	*eventually before*		$\Diamond$	*eventually afterward*
	$\blacksquare$	*always before*		$\Box$	*always afterward*
	$\widehat{\bullet}$	*weak previous*		$\widehat{\circ}$	*weak next*

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past	`<`	*previous*	*future*	`>`	*next*
	`<?`	*since*		`>?`	*until*
	`<*`	*trigger*		`>*`	*release*
	`<?`	*eventually before*		`>?`	*eventually afterward*
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  - Boolean operators `&` `|` `~`

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- `&initial` $\mathbf{I}$
- `&final` $\mathbb{F}$
- `&tel { φ }` for temporal formula φ
- Example

$$\mathit{shoot} \wedge \blacksquare \mathit{unloaded} \wedge \bullet\blacklozenge \mathit{shoot} \rightarrow \bot$$

can be expressed as

```
:- shoot , &tel { <* unloaded & < <? shoot }.
```

or

```
:- &tel { shoot & <* unloaded & < <? shoot }.
```

Wolf, sheep, and cabbage

```
#program always.

item(w;s;c).
opp(l,r). opp(r,l).
eats(w,s). eats(s,c).

#program initial.

at(b,l).
at(X,l) :- item(X).                          % everything at the left bank

#program dynamic.

at(X,A) :- 'at(X,B), m(X), opp(A,B).         % effect axiom for moving item X

at(b,A) :- 'at(b,B), opp(A,B).               % boat is always moving

at(X,A) :- 'at(X,A), not at(X,B), opp(A,B).  % inertia

0 { m(X) : item(X) } 1.                      % choose moving at most one item

#program always.

:- m(X), 'at(b,A), 'at(X,B), opp(A,B).       % we cannot move item X if at the opposite bank

:- eats(X,Y), at(X,A), at(Y,A), opp(A,B), at(b,B).  % we cannot leave them alone

#program final.

:- at(X,l).

#show m/1.
```

telingo's solution

```
$ telingo version 1.0
Reading from wolf.tel
Solving...
Solving...
Solving...
Solving...
Solving...
Solving...
Solving...
Solving...
Answer: 1
 State 0:
 State 1:  m(s)
 State 2:
 State 3:  m(w)
 State 4:  m(s)
 State 5:  m(c)
 State 6:
 State 7:  m(s)
Answer: 2
 State 0:
 State 1:  m(s)
 State 2:
 State 3:  m(c)
 State 4:  m(s)
 State 5:  m(w)
 State 6:
 State 7:  m(s)
SATISFIABLE

Models       : 2
Calls        : 8
Time         : 0.156s (Solving: 0.00s)
CPU Time     : 0.028s
```

Outline

Summary

- *TEL_f*
  - combines *HT* and *LTL* on finite traces
  - reducible to a normal form close to logic programs
  - naturally accounts for dynamic KRR
  - advocates past temporal operators
  - offers embeddings for action languages
  - readily implementable via ASP
- ASP-based systems for *TEL_f*
  - `https://github.com/potassco/telingo`
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- What's next? Linear dynamic ASP (cf. forthcoming KR'18 paper)
  - extension of *TEL_f*
  - offers Golog-style control